

# Valuing Private Equity Investments Strip by Strip

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NYU STERN

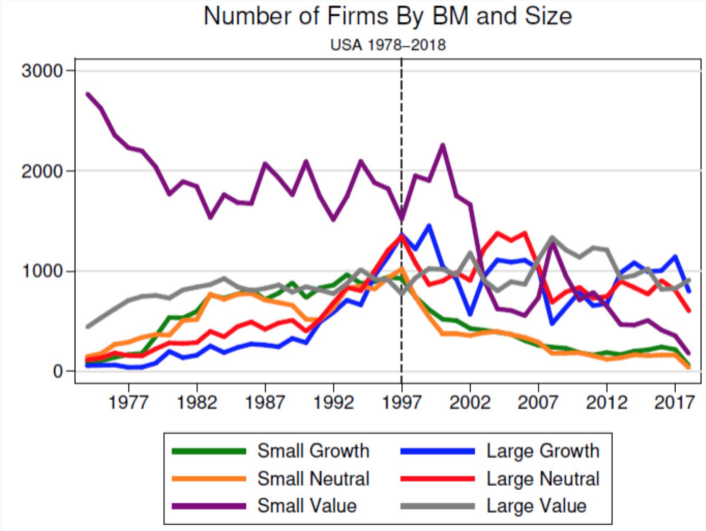
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UNC

# Private Universe is Expanding vs Public Stocks, Complicating Risk Assessment



## Measurement Problems in Assessment of Risk and Return in Private Equity

1. **The shift in capital towards private markets, especially for small growth firms, means that we need to know how risk is priced in private markets**

# Measurement Problems in Assessment of Risk and Return in Private Equity

1. The shift of capital towards private markets, especially for small growth firms, means that we need to know how risk is priced in private markets
2. **Existing approaches to private equity valuation have not taken into account the multivariate nature of risk nor the temporal composition of risk**
  - Standard approaches:
    - TVPI (no discounting, no risk)
    - IRR (no risk)
    - PME (beta = 1)
    - GPME (beta constant)
  - Limitations to all approaches: only one aggregate source of risk
    - If a bad assumption in equities (CAPM fails), why good assumption in PE?
    - Especially in “alternative” categories like Real Estate Funds

# Measurement Problems in Assessment of Risk and Return in Private Equity

1. Shift to private markets
2. Literature has struggled with cross-section and term structure of risk
3. **To address these limitations, we draw from asset pricing literature emphasizing rich *cross-section* of factors and *term structure* of risk**
  - Multifactor models: Fama and French (2016, 2018) and Hou, Xue, and Zhang (2015, 2017, 2018) and a budding literature on machine learning and the XS (e.g., Kozak, Nagel, and Santosh, 2017, Gu, Kelly, and Xiu, 2018)
  - Term structure: van Binsbergen, Brandt, and Koijen (2012), van Binsbergen and Koijen (2017)
    - Term structure of risk potentially upward or downward sloping, depending on factor
    - Term structure of strips on other factors not known or traded

# Measurement Problems in Assessment of Risk and Return in Private Equity

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4. **Problem: Observe cash flows, not returns**

# Measurement Problems in Assessment of Risk and Return in Private Equity

1. Shift to private markets
2. Literature has struggled with cross-section and term structure of risk
3. We draw from other asset pricing literature emphasizing multifactor models
4. Problem: Observe cashflows, not returns
5. **Our Solution:**
  - Estimate exposures of PE fund cash flows to cash flows on bond and cross-section of stock strips
  - Use asset pricing model to price these strips

**Delivers PE factor exposure, expected return, risk-adjusted profit, NAV**

1. **Alternative PE Categories have sector-specific factor loadings in the cross-section**



## Key Takeaways

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2. **Substantial small and growth factor exposure, suggesting these factor loadings are prominent in PE**

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2. Substantial small and growth factor exposure, suggesting these factor loadings are prominent in PE
3. **Decreasing realized profits and expected returns in more recent vintages**

## Key Takeaways

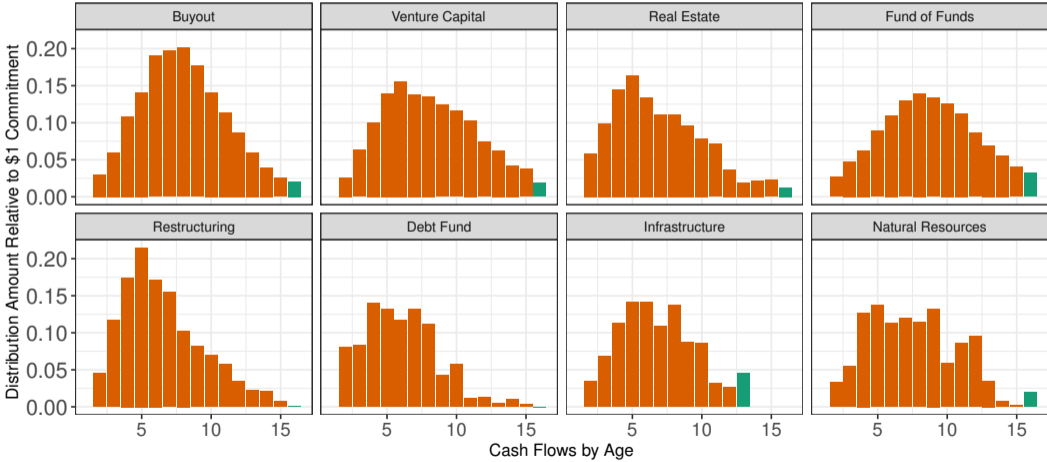
1. Alternative PE Categories have sector-specific factor loadings in the cross-section
2. Decreasing realized profits and expected returns in more recent vintages
3. Substantial small and growth factor exposure, suggesting these factor loadings are prominent in PE
4. **Rich risk adjustment results in much lower average risk-adjusted profits across all fund categories**

1. Estimate Private Equity **fund exposure** to public market assets
2. **Price public market assets.** Model delivers term structure of risk in bond and stock market, including for cross-sectional risk factors
3. Value the **replicating portfolios** of PE fund exposures using the bond and stock strip prices
4. Use asset pricing model to understand private equity **risk/return characteristics and risk-adjusted profits**

# 1. Private Equity Fund Exposures

Estimate fund exposure to multi-factor model

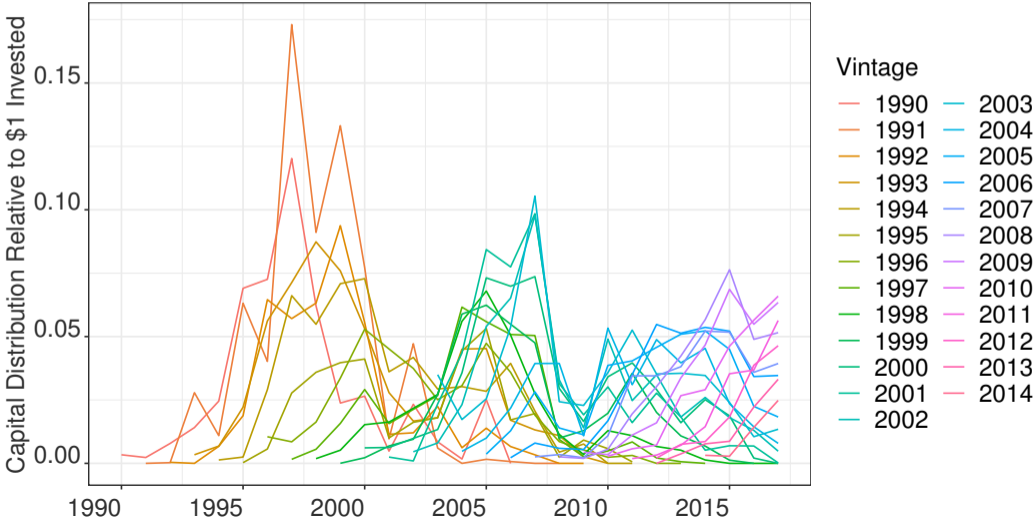
# Want to Understand Cash-Flow Profiles of Private Equity Funds



Future Cash Flow Actual Cash Flow

# Cash-Flow Variation Across Horizon and Vintage — Buyout

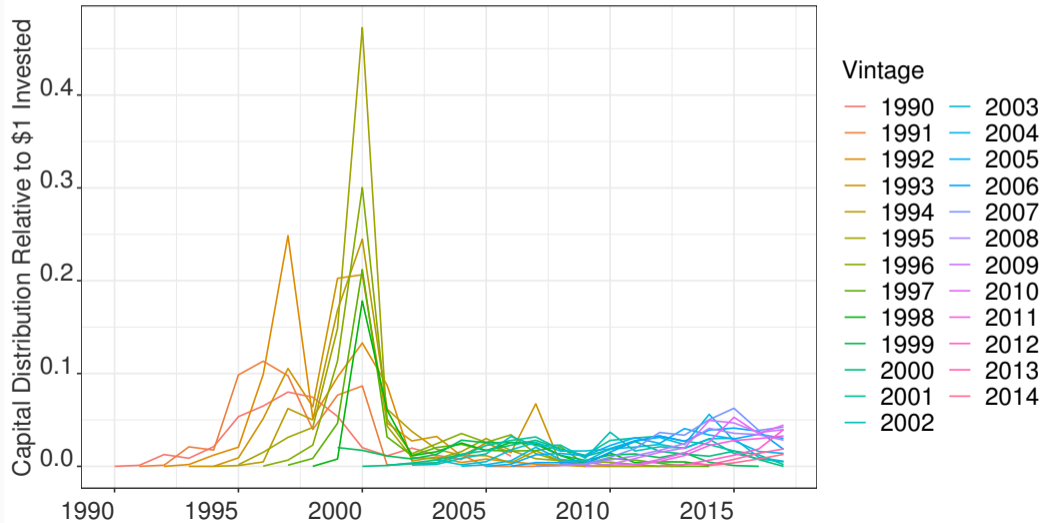
Real Estate    Infrastructure



# Cash-Flow Variation Across Horizon and Vintage – VC

Real Estate

Infrastructure





# Break out Factor Exposure Strip by Strip to Match PE Cash Distributions to LPs

- Construct  $F_{t,t+h}$  cash flows on replicating portfolio:
  - Bond strips, pay \$1 for sure at horizon  $t + h$
  - Dividend strips, pay one risky cash flow at  $t + h$ :  $D_{t+h}/D_t$
  - Gain strips, pays one risky cash flow at  $t + h$ :  $P_{t+h}/P_t$
- $h$ -period stochastic discount factor chains one-period SDFs:

$$M_{t,t+h} = \prod_{k=1}^h M_{t+k}$$

- Defining prices of these strips:

$$P_{t,h} = \mathbb{E}_t[M_{t,t+h}F_{t,t+h}]$$

## Estimate Factor Exposure Strip-by-Strip

- $X_{t+h}^i$  is the distribution cash flow of PE fund  $i$  of vintage  $t$  at time  $t + h$
- Factor model fitting quarterly PE fund cash flows with quarterly strip cash flows

$$X_{t+h}^i = \beta_{t,h}^i \mathbf{F}_{t,t+h} + e_{t+h}^i$$

Factors  $\mathbf{F}$  are:

- Zero Coupon Bond (constant)
- Dividend Strips: market index, small, growth, value, REIT, infrastructure, and natural resource stocks
- Gain Strips: market index, small, growth, value, REIT, infrastructure, and natural resource stocks
- 15 factors at each horizon  $h$

# Shrinkage Estimators Measure Factor Exposure

Structure of exposure (e.g., Buyout funds):

$$\begin{aligned}X_{t+h}^{i \in c} &= \beta_{t,h}^{bond} + \beta_{t,h}^{equity} F_{t,t+h}^{equity} + e_{t+h}^i \\ &= a_{pd_t}^{bond} + b_h^{bond} + \left( a_{pd_t}^{equity} + b_h^{equity} \right) F_{t,t+h}^{equity} + e_{t+h}^i.\end{aligned}$$

Allow  $b_h$  to vary for each horizon (year)

$a_t$  varies for each quartile of the P/D distribution of vintage quarter

Two estimation techniques: Validation on Public Equities

1. OLS (only stock market dividend strips and zero coupon bonds)
2. Elastic Net:

$$\hat{\beta}_{ElasticNet} = \arg \min_{\beta \in \mathbf{R}^{KH}} \|X_{t+h}^i - \beta_{t,h}^i F_{t,t+h}\|_2^2 + \lambda_0 \mathbf{1}\{\beta < \mathbf{0}\} + \lambda_1 \|\beta\|_1 + \lambda_2 \|\beta\|^2, \quad \lambda_0 = \infty$$

## Estimation Enables Novel Understanding of PE Asset Pricing

- Use model to understand expected returns, where  $\mathbf{w}_{t,h}^k = \beta_{t,h}^k \mathbf{P}_{t,h}$

$$\mathbb{E}_t [R^i] = \sum_{h=1}^H \sum_{k=1}^K \mathbf{w}_{t,h}^k \mathbb{E}_t [R_{t+h}(k)]$$

- Profit corrects for risk (may include premium for illiquidity):

$$\begin{aligned} e_{t+h}^i &= X_{t+h}^i - \beta_{t,h}^i \mathbf{F}_{t,t+h} \\ \text{RAP}_t^i &= \underbrace{\left( \sum_{h=1}^H X_{t+h}^i P_{t,h}^\$ - 1 \right)}_{\sim \text{TVPI}} - \underbrace{\left( \sum_{h=1}^H \sum_{k=1}^K \beta_{t,h}^k \mathbf{F}_{t,t+h} P_{t,h}^\$ - \beta_{t,h}^k \mathbf{P}_{t,h} \right)}_{\text{Risk adjustment}} \\ &= \sum_{h=1}^H e_{t+h}^i P_{t,h}^\$ + \sum_{h=1}^H \sum_{k=1}^K \beta_{t,h}^k \mathbf{P}_{t,h} - 1 \end{aligned}$$

## 2. Price Public Assets

Asset pricing model fits capital market assets

# Asset Pricing Model

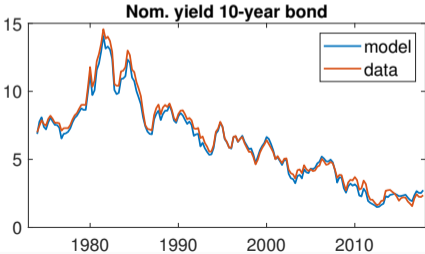
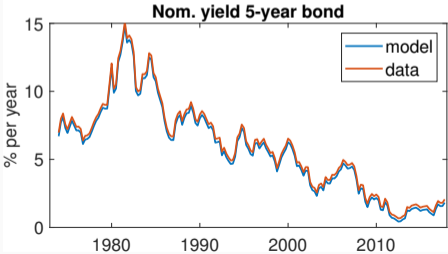
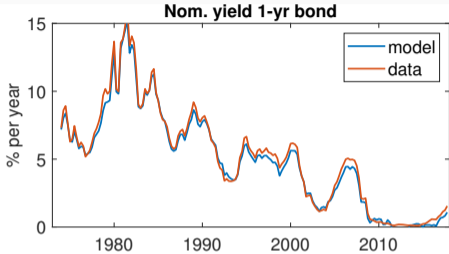
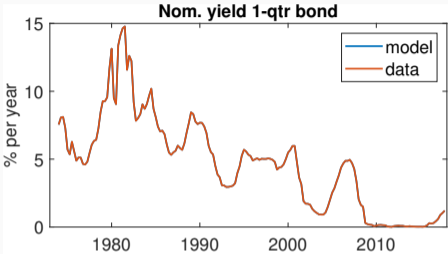
- State variables follow Gaussian first-order VAR:

$$\mathbf{z}_t = \Psi \mathbf{z}_{t-1} + \Sigma^{\frac{1}{2}} \varepsilon_t, \quad \varepsilon_t \sim i.i.d. \mathcal{N}(0, I)$$

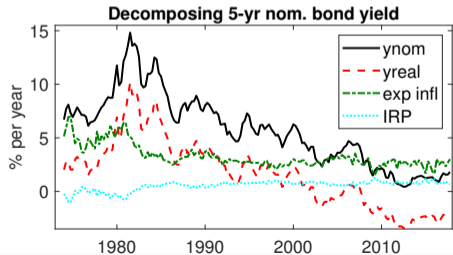
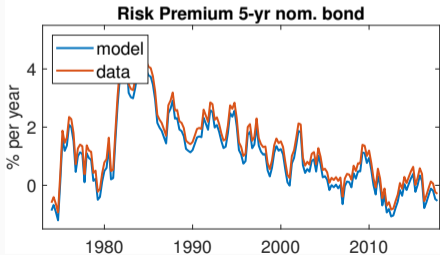
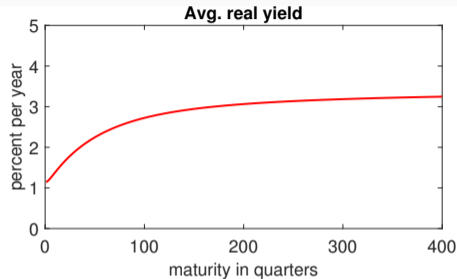
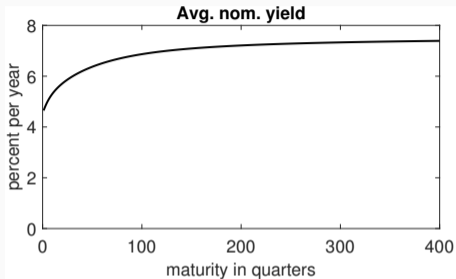
- Bond variables: inflation, real GDP growth, nominal short rate, five year - three month Treasury yield spread
- Stock variables: log price-dividend ratio and log real dividend growth for: CRSP vw-aggregate market, NAREIT real estate, listed infrastructure, small stocks (ME Q1), growth stocks (BM Q1), value stocks (BM Q5), natrl. resources
- Log SDF:

$$\begin{aligned} m_{t+1}^{\$} &= -y_t^{\$(1)} - \frac{1}{2} \Lambda_t' \Lambda_t - \Lambda_t' \varepsilon_{t+1} \\ \Lambda_t &= \Lambda_0 + \Lambda_1' z_t \end{aligned}$$

# Model Matches Time-Series of Bond Yields

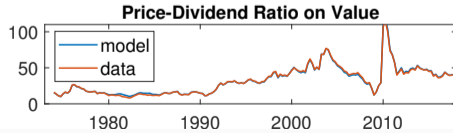
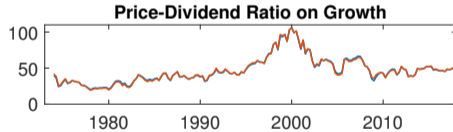
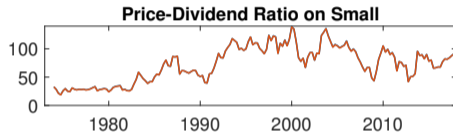
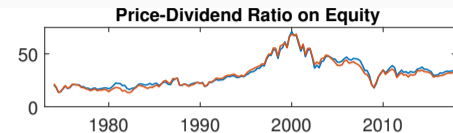
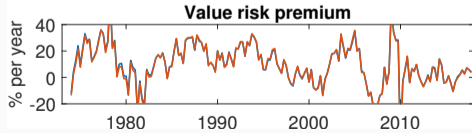
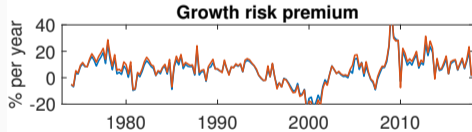
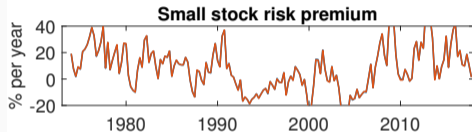
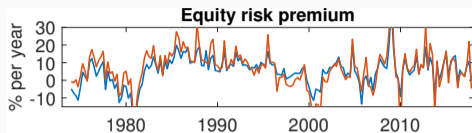


# Also Matches underlying Components of Bond Yield: Real + Nominal

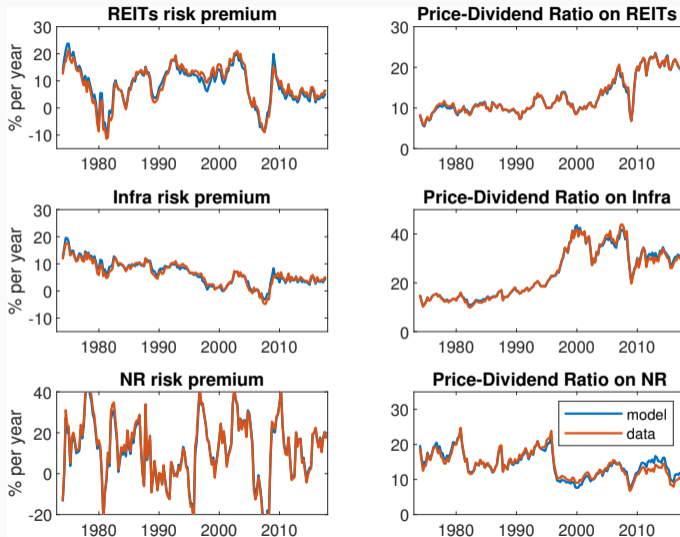




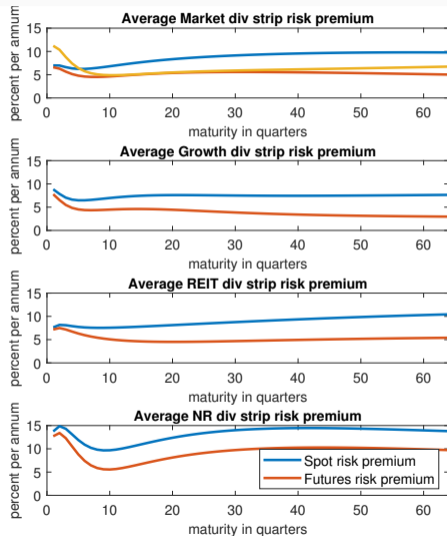
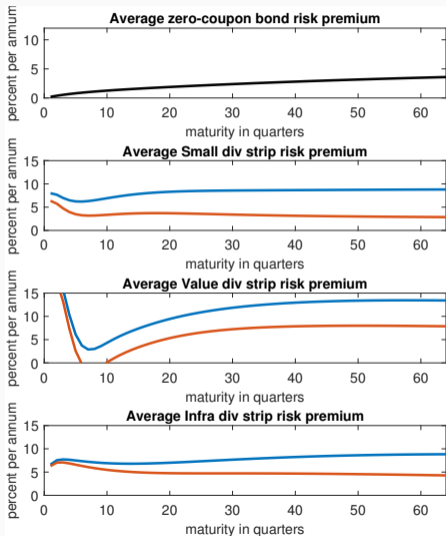
# Fits Equity Risk Premia as well as Stock Price Levels



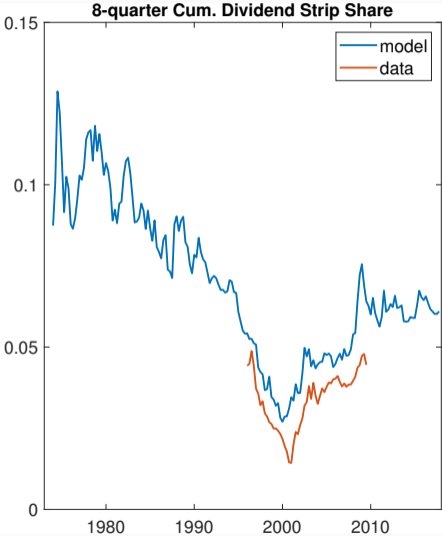
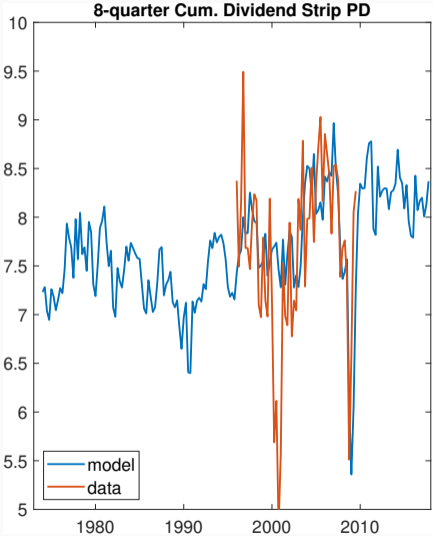
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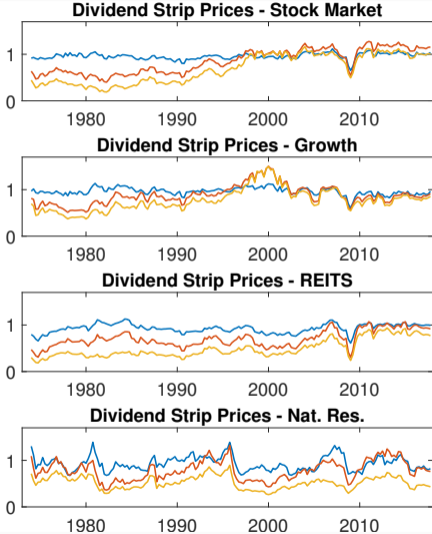
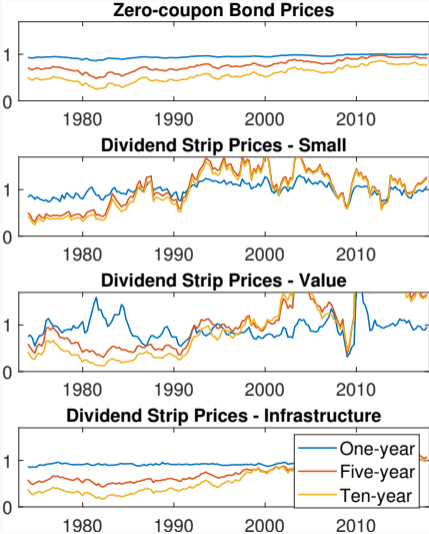
# Rich Patterns in Temporal Pricing of Risk



# Imputed Dividend Strip Model Matches Data when Available



# Outcome of Model: Bond + Dividend Strip Prices

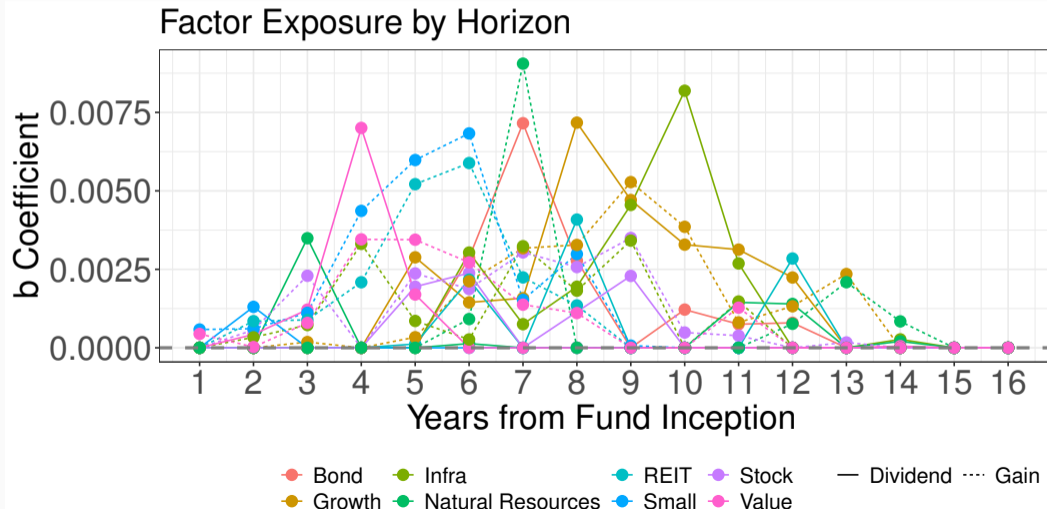


# 3. Replicating Portfolios

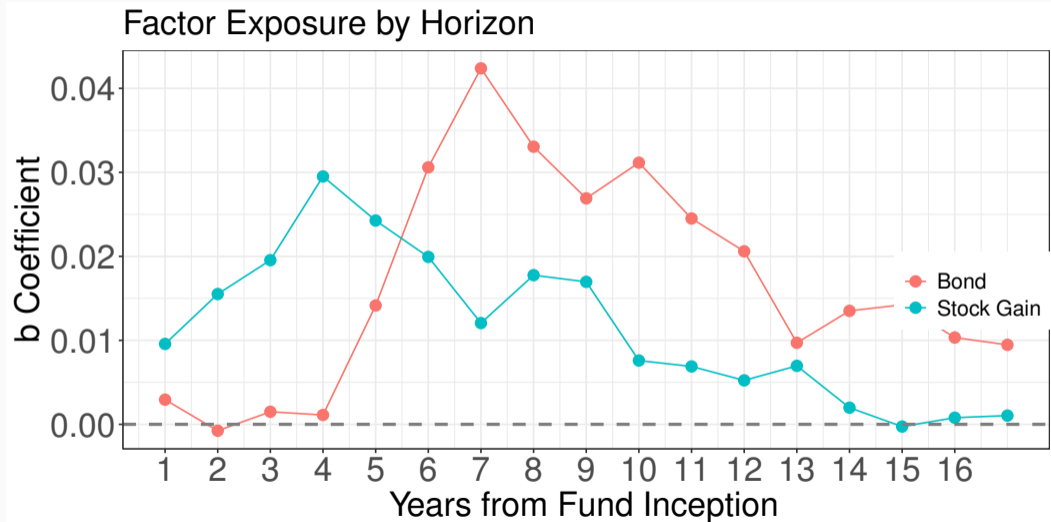
Construct cash-flow replicating portfolios for Private Equity Funds

# Factor Exposure in PE Funds by Horizon — Buyout, Elastic Net

Buyout - P/D

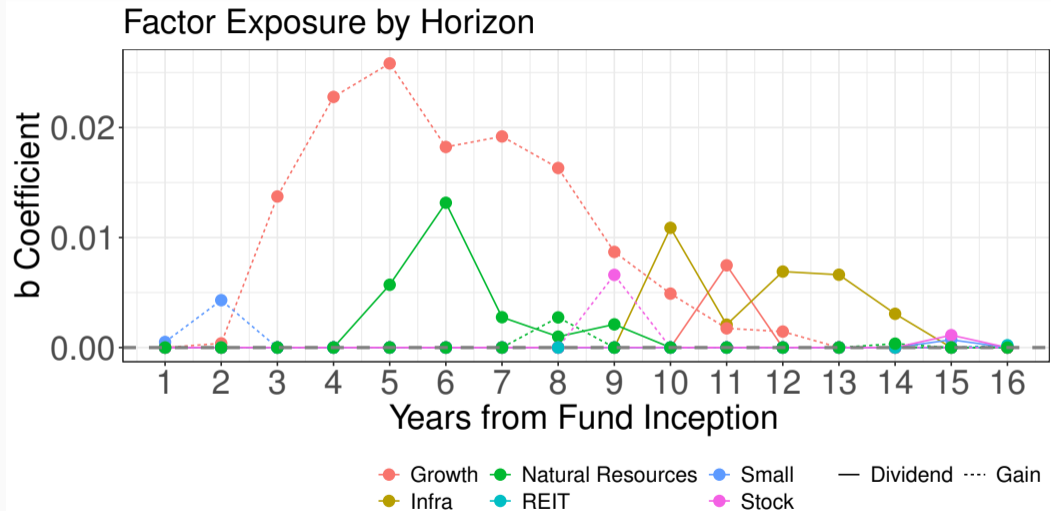


# Factor Exposure in PE Funds by Horizon — Buyout, OLS



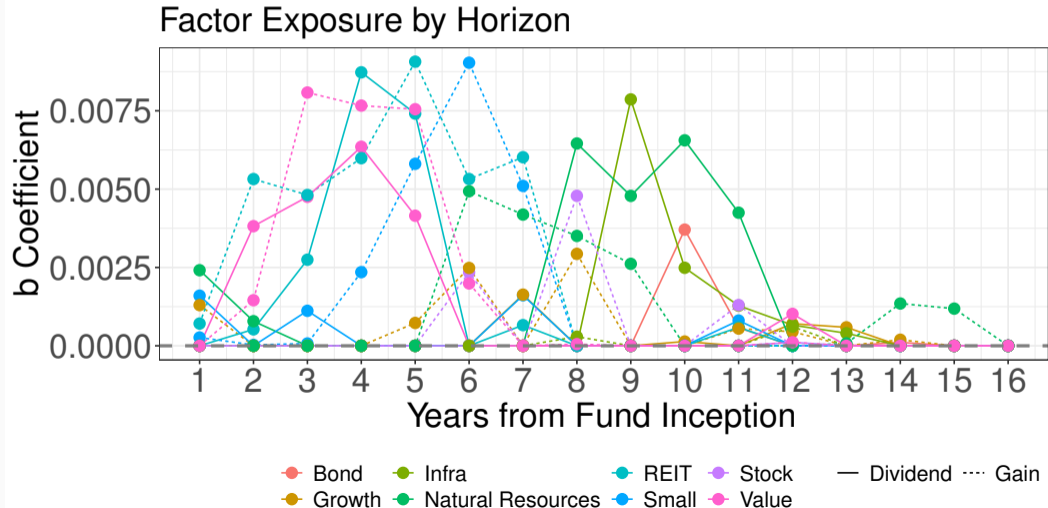


# Factor Exposure in PE Funds by Horizon – VC, Elastic Net



# Factor Exposure in PE Funds by Horizon — Real Estate, Elastic Net

Infrastructure



# 4. Private Equity Fund Characteristics

Use fitted model to understand risk and characteristics of private equity funds

## Model Comparison

	Buyout		VC		Real Estate		Fund of Funds	
	Mean	St Dev	Mean	St Dev	Mean	St Dev	Mean	St Dev
TVPI	0.62	(0.74)	0.39	(1.69)	0.17	(0.52)	0.23	(0.51)
IRR (%)	0.09	(0.10)	0.03	(0.20)	0.04	(0.11)	0.05	(0.07)
PME-1	0.36	(0.67)	0.22	(1.49)	-0.04	(0.44)	0.17	(0.40)
RAP 2-factor (NPV Call)	0.28	(0.53)	-0.15	(1.36)	0.09	(0.45)	0.24	(0.50)
RAP 15-factor (NPV Call)	-0.06	(0.51)	-0.09	(1.27)	-0.16	(0.38)	-0.19	(0.35)
RAP 2-factor (Sum Call)	0.20	(0.53)	-0.25	(1.36)	0.04	(0.45)	0.15	(0.51)
RAP 15-factor (Sum Call)	-0.14	(0.51)	-0.18	(1.27)	-0.20	(0.38)	-0.28	(0.36)

## Model Comparison, Burgiss

	Restructuring		Debt Fund		Infrastructure		Natural Resources	
	Mean	St Dev	Mean	St Dev	Mean	St Dev	Mean	St Dev
TVPI	0.44	(0.57)	0.30	(0.27)	0.17	(0.65)	0.33	(0.91)
IRR (%)	0.09	(0.10)	0.07	(0.04)	0.03	(0.11)	0.02	(0.18)
PME-1	0.20	(0.56)	0.12	(0.17)	0.17	(0.57)	0.28	(0.86)
RAP 2-factor (NPV Call)	0.17	(0.47)	0.34	(0.55)	0.33	(0.65)	0.07	(0.66)
RAP 15-factor (NPV Call)	-0.001	(0.46)	-0.13	(0.30)	-0.06	(0.58)	-0.06	(0.62)
RAP 2-factor (Sum Call)	0.13	(0.47)	0.31	(0.56)	0.27	(0.66)	0.00	(0.65)
RAP 15-factor (Sum Call)	-0.04	(0.46)	-0.16	(0.31)	-0.12	(0.59)	-0.13	(0.60)

# PE Fund Risk-Adjusted Profits – Buyout, Elastic Net

Buyout - Elastic Net

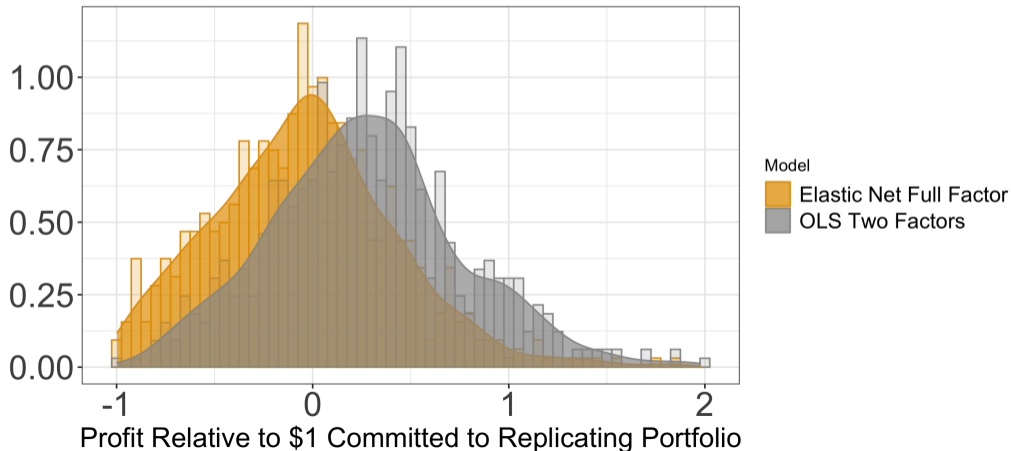
Venture Capital - Elastic Net

Real Estate - Elastic Net

Infrastructure - Elastic Net

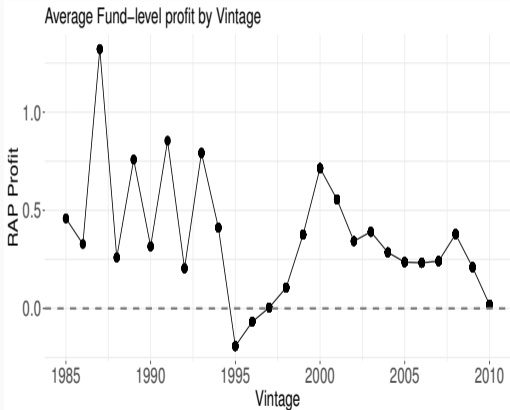
## Histogram of Fund-Level Profit Relative to Replicating Portfolio

TVPI is:0.622 Risk-Adj Profit is:-0.063 Risk Adj Fraction above 10% is:0.346

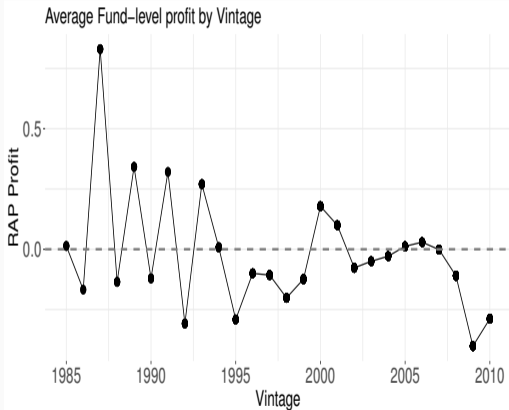


# PE Fund Risk-Adjusted Profits – Buyout, OLS + Elastic Net

## 2-factor OLS Model

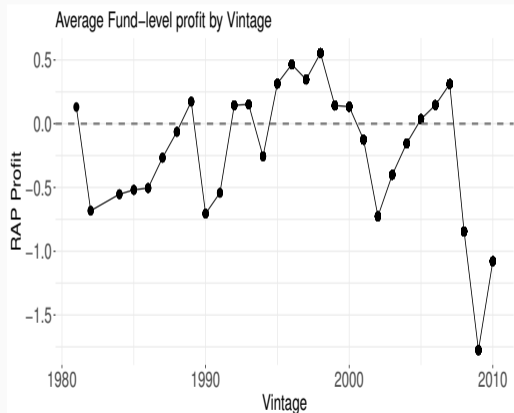


## Full-factor Elastic Net Model

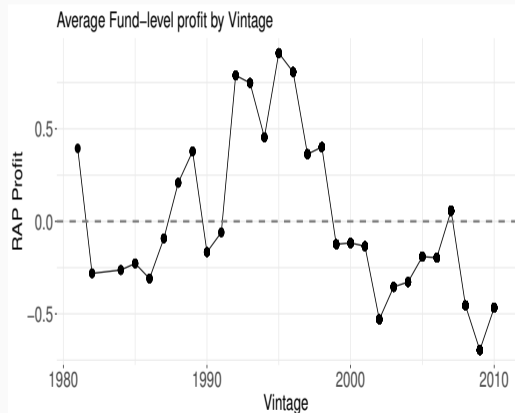


# VC Fund Risk-Adjusted Profits – Buyout, OLS + Elastic Net

## 2-factor OLS Model



## Full-factor Elastic Net Model

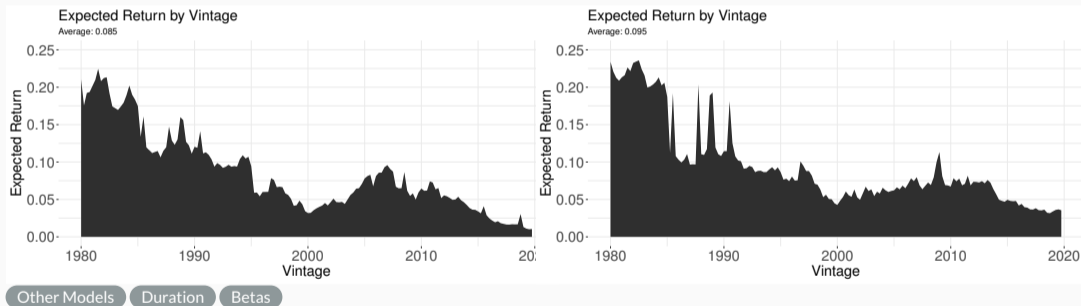




# PE Expected Return – Buyout, OLS + Elastic Net

## 2-factor OLS Model

## Full-factor Elastic Net Model



# PE Comparison with PME – Buyout

Buyout

Venture Capital

Real Estate

Infrastructure

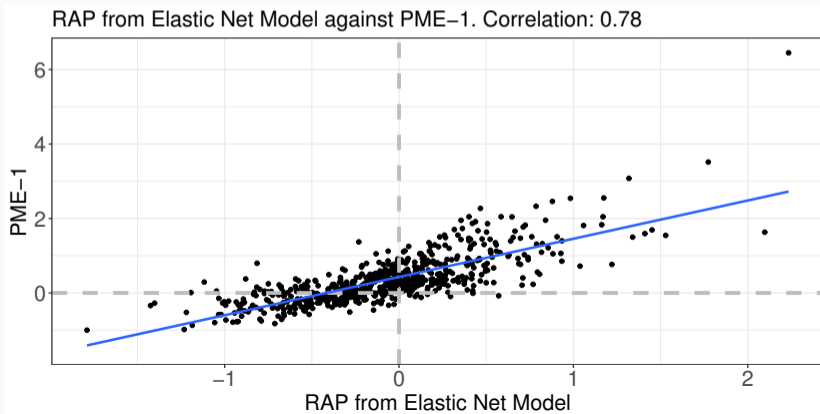
Buyout - Elastic Net

Venture Capital - Elastic Net

Real Estate - Elastic Net

Infrastructure - Elastic Net

IRR Comparison



## Takeaways

1. Develop methodology to value and understand risk/return characteristics when only cash flows, not returns, are available
2. Find PE funds take asset-specific specific exposure. Small, growth, real estate, infra exposure has migrated to PE
3. Risk-adjusted profit, as well as expected return on replicating portfolios, declining over time
4. Lower and less persistent RAP suggest that LPs do not earn abnormal returns; maybe even illiquidity premium for convenience of not marking to market

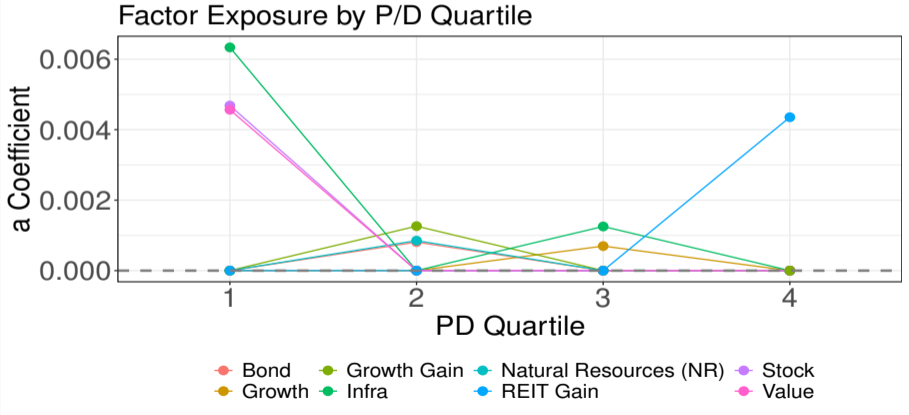
Dividend strip data:

[https://github.com/arpitrage/Dividend\\_Strip](https://github.com/arpitrage/Dividend_Strip)

# Factor Exposure in PE Funds by P/D Ratio – Buyout, Elastic Net

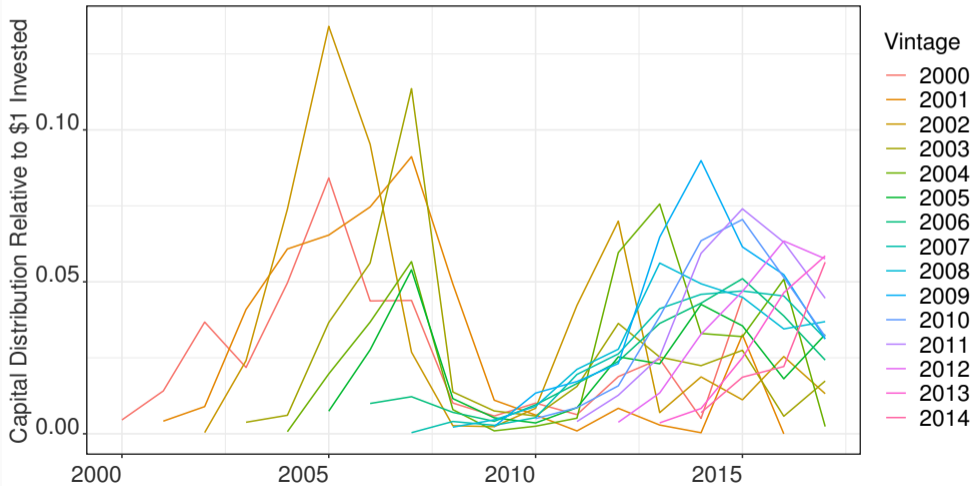
Back Buyout Venture Capital Real Estate Infrastructure

Buyout - Elastic Net Venture Capital - Elastic Net Real Estate - Elastic Net Infrastructure - Elastic Net



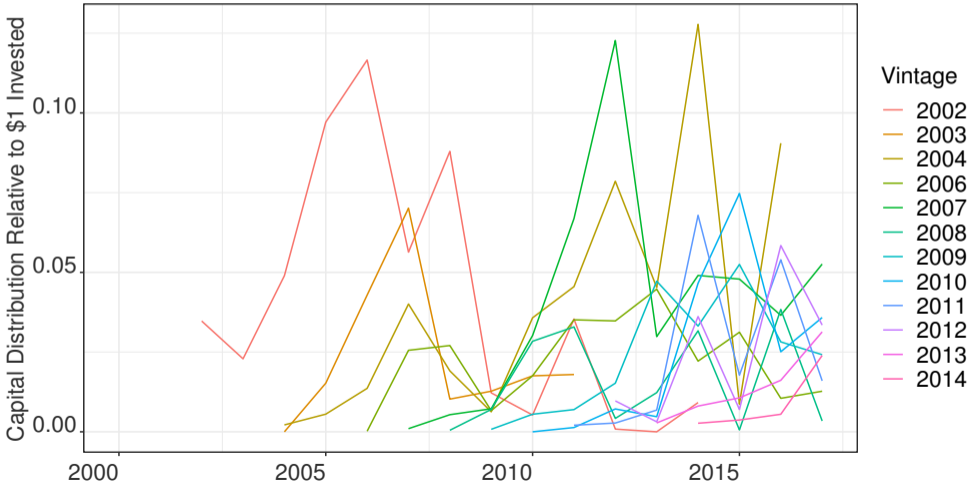
# Cash-Flow Variation Across Horizon and Vintage — Real Estate

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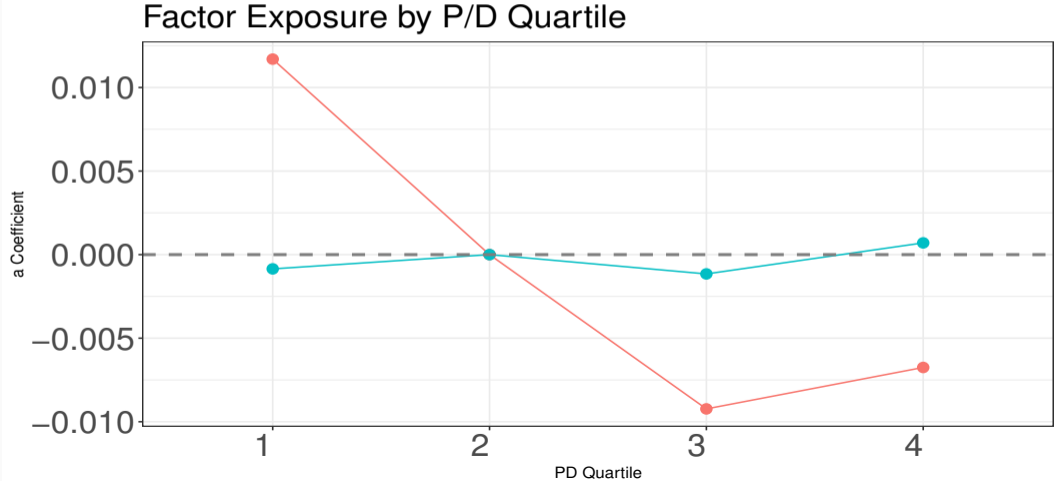
# Cash-Flow Variation Across Horizon and Vintage — Infrastructure

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# Factor Exposure in PE Funds by P/D Ratio – Buyout

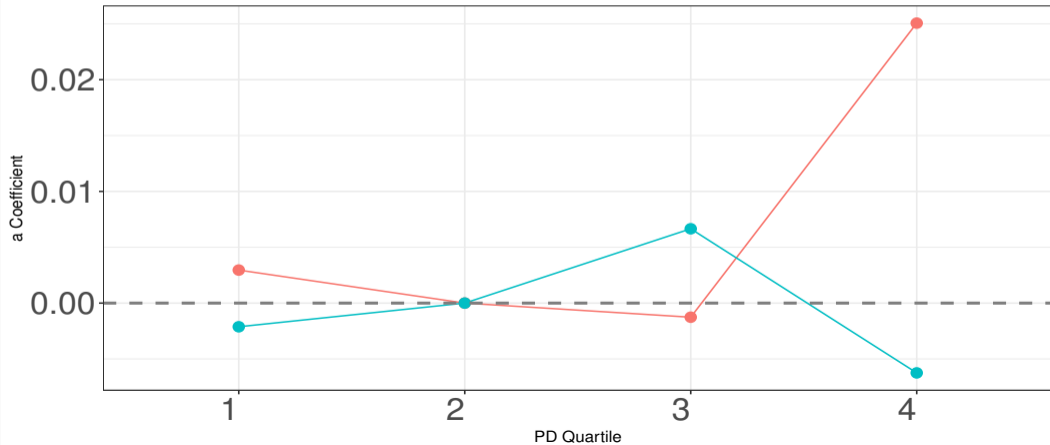
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# Factor Exposure in PE Funds by P/D Ratio – VC

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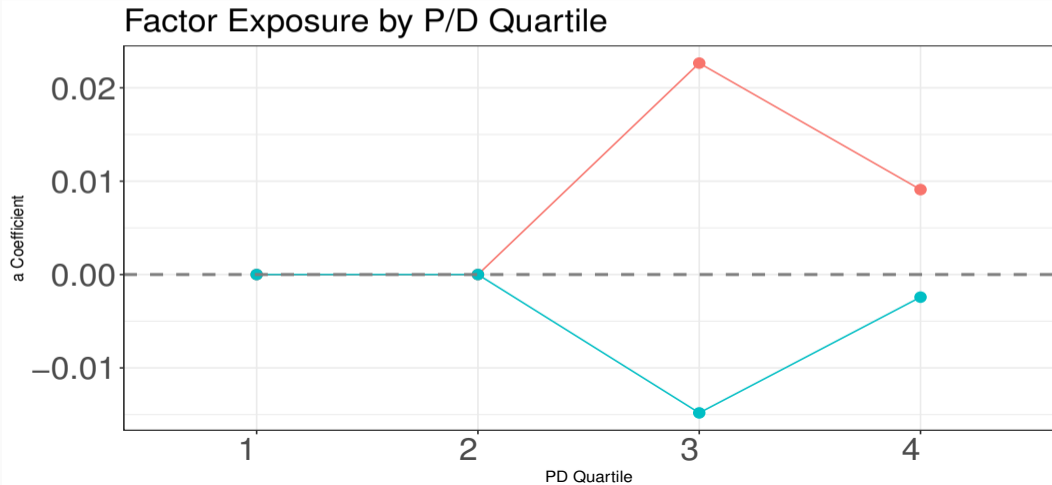
## Factor Exposure by P/D Quartile





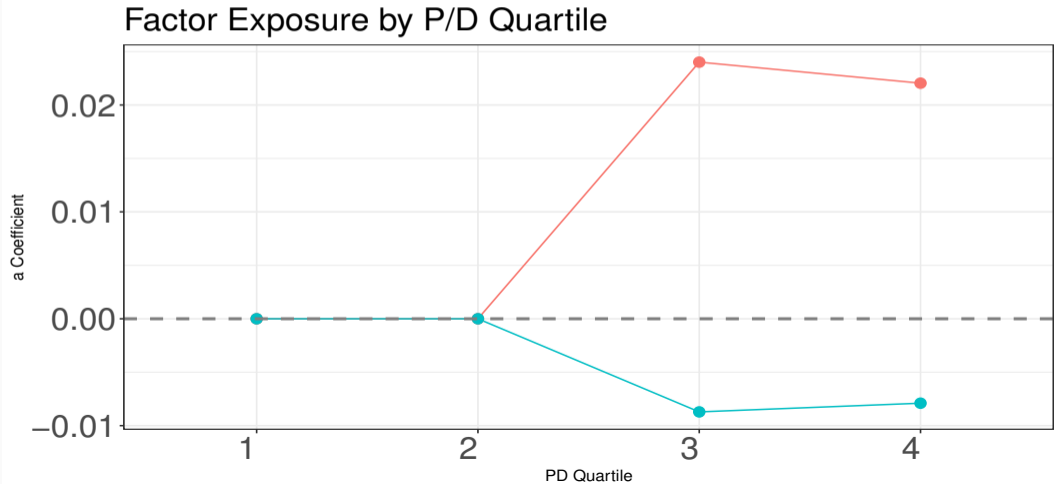
# Factor Exposure in PE Funds by P/D Ratio — Real Estate

Back



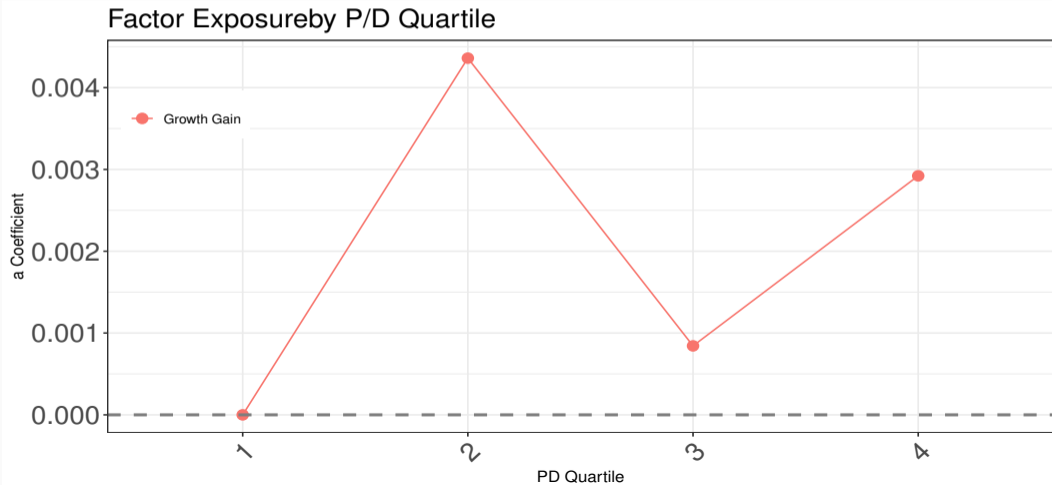
# Factor Exposure in PE Funds by P/D Ratio – Infrastructure

Back



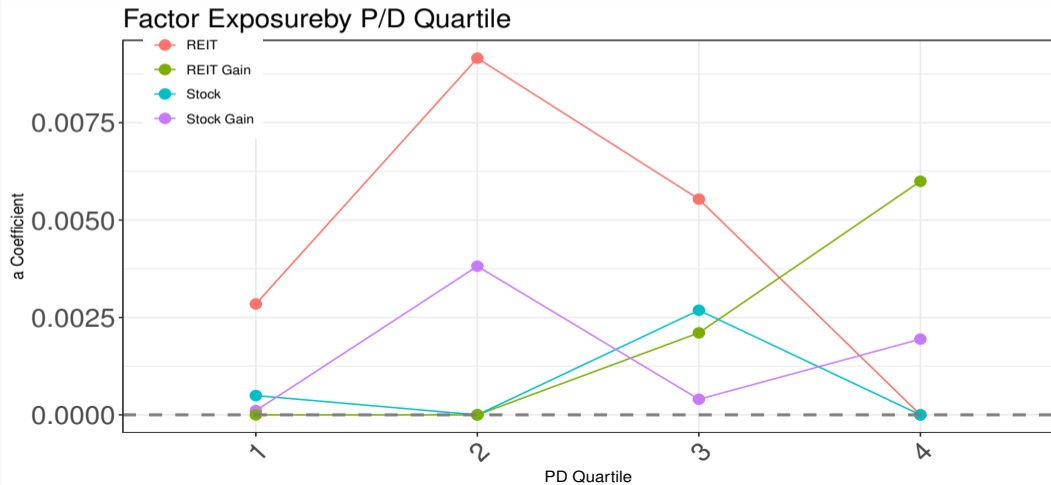
# Factor Exposure in PE Funds by P/D Ratio – VC, Elastic Net

Back



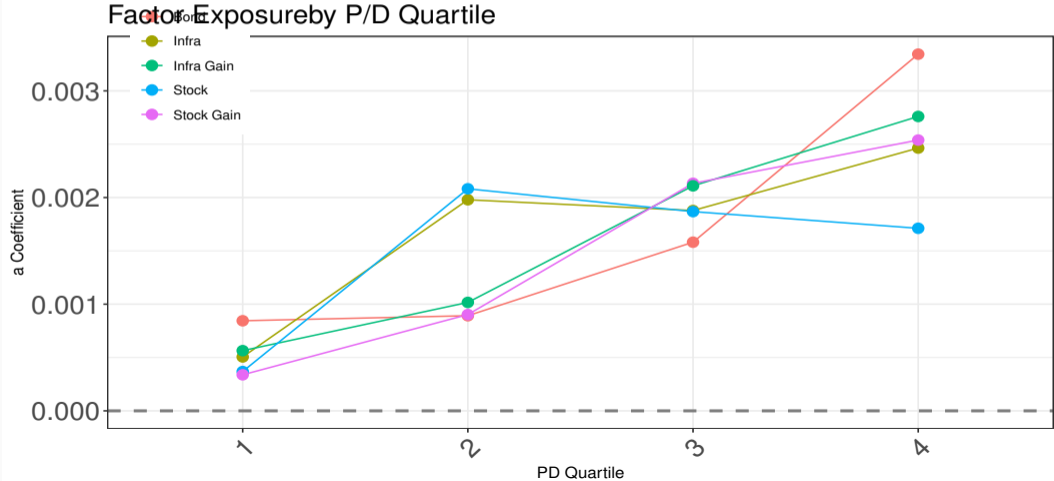
# Factor Exposure in PE Funds by P/D Ratio – Real Estate, Elastic Net

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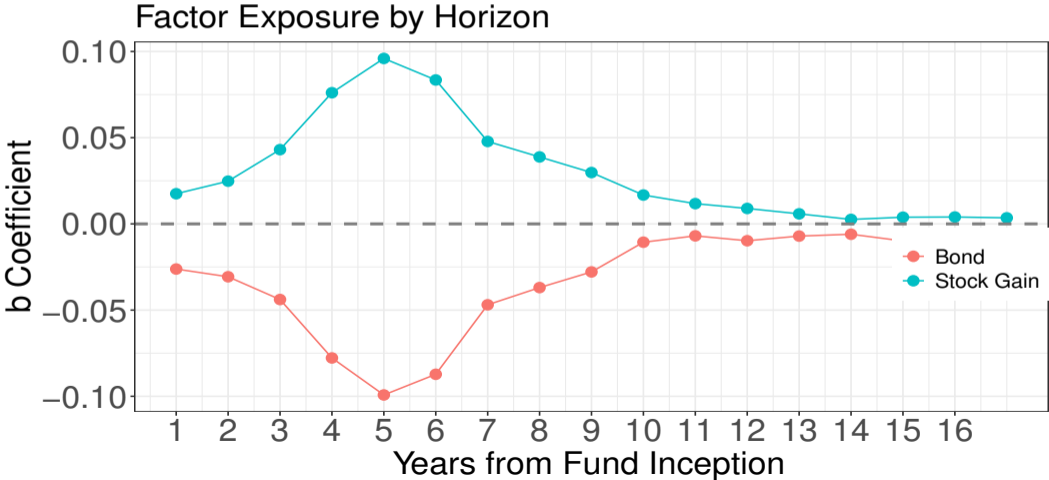
# Factor Exposure in PE Funds by P/D Ratio – Infrastructure, Elastic Net

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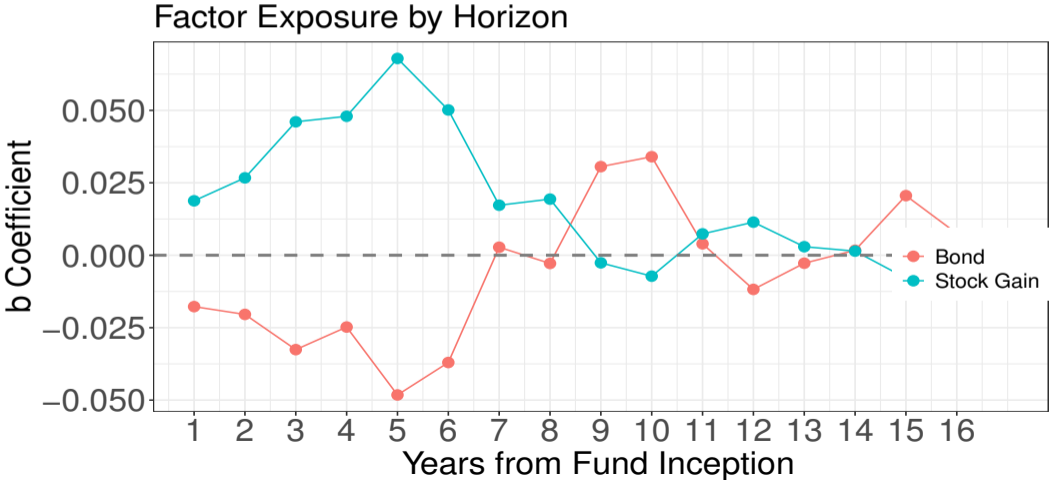
# Factor Exposure in PE Funds by Horizon – VC, OLS

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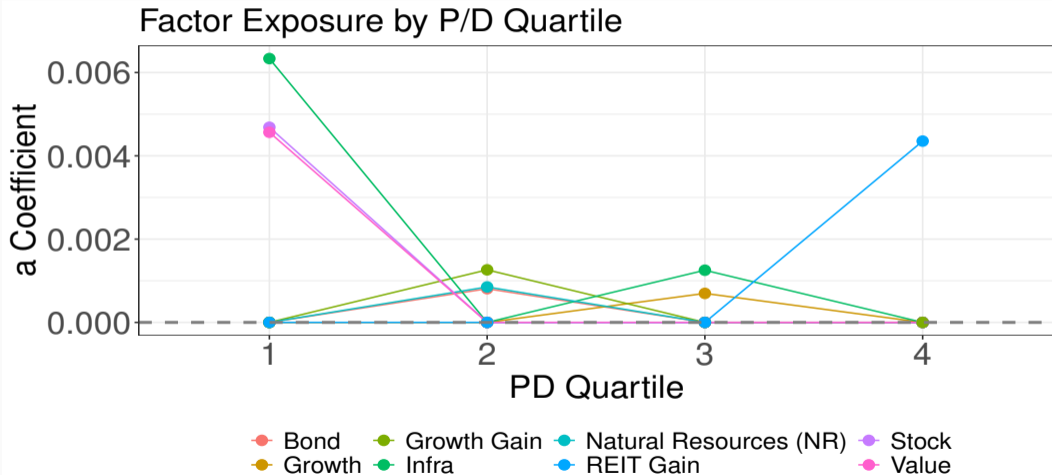


# Factor Exposure in PE Funds by Horizon — Real Estate, OLS

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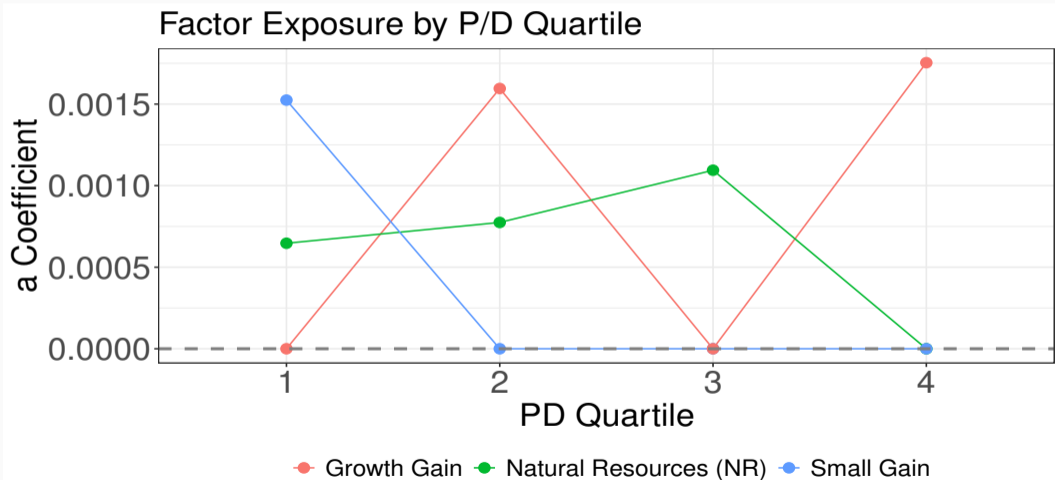


# Factor Exposure in PE Funds by Horizon – Buyout, Elastic Net

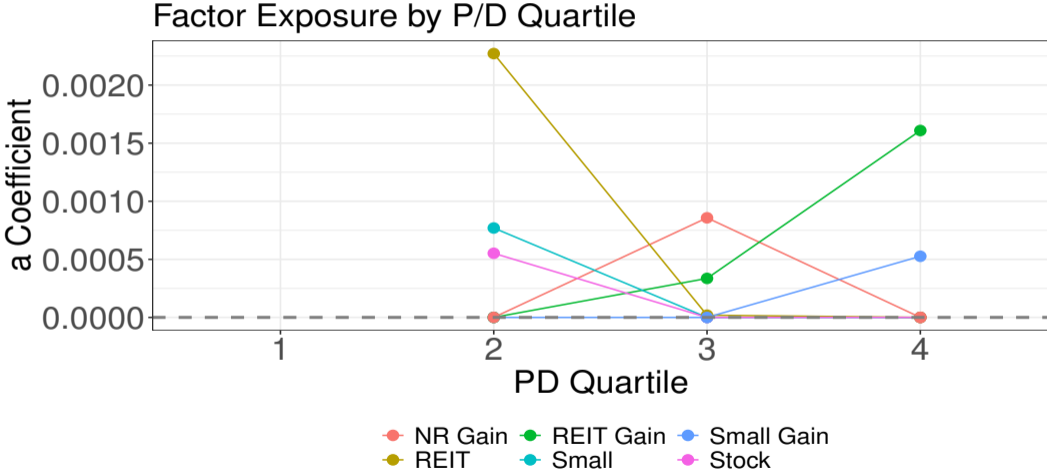




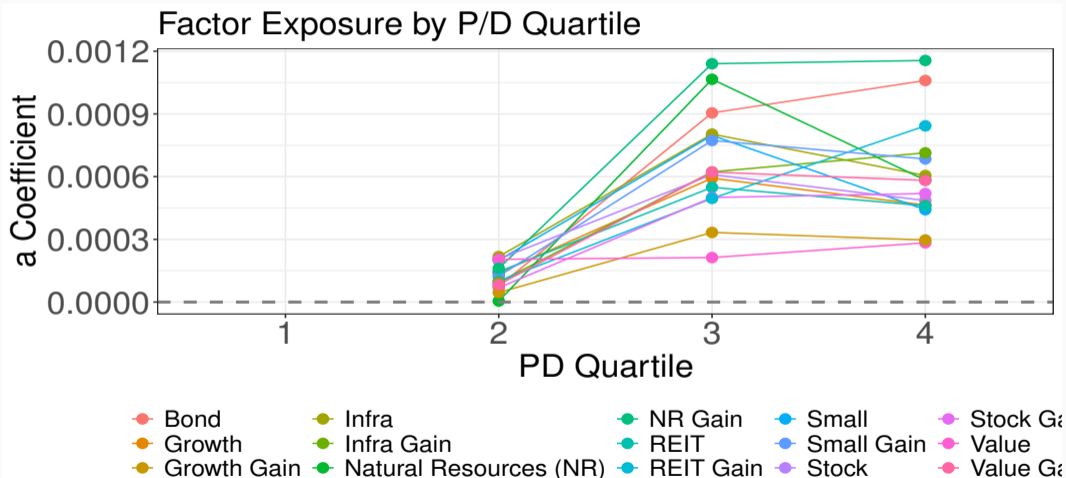
## Factor Exposure in PE Funds by Horizon – VC, Elastic Net



# Factor Exposure in PE Funds by Horizon – Real Estate, Elastic Net



# Factor Exposure in PE Funds by Horizon – Infrastructure, Elastic Net

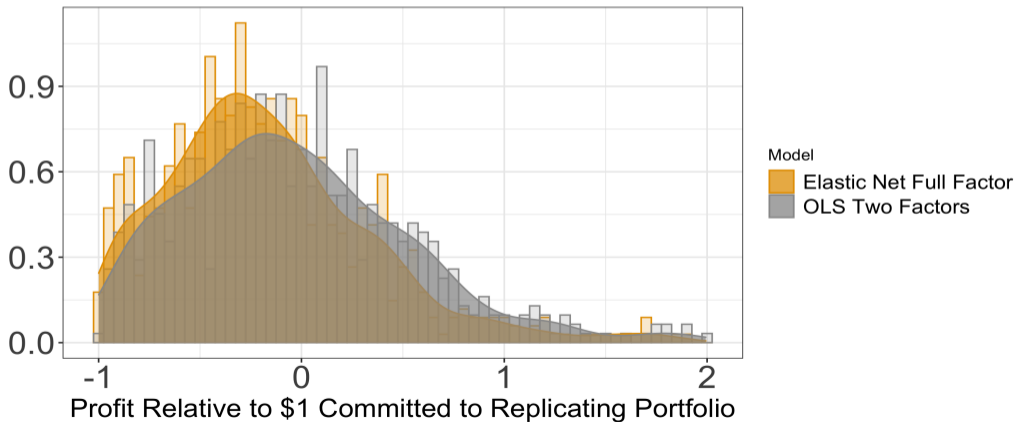


# PE Fund Risk-Adjusted Profits – VC, Elastic Net

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## Histogram of Fund-Level Profit Relative to Replicating Portfolio

TVPI is:0.394 Risk-Adj Profit is:-0.088 Risk Adj Fraction above 10% is:0.263

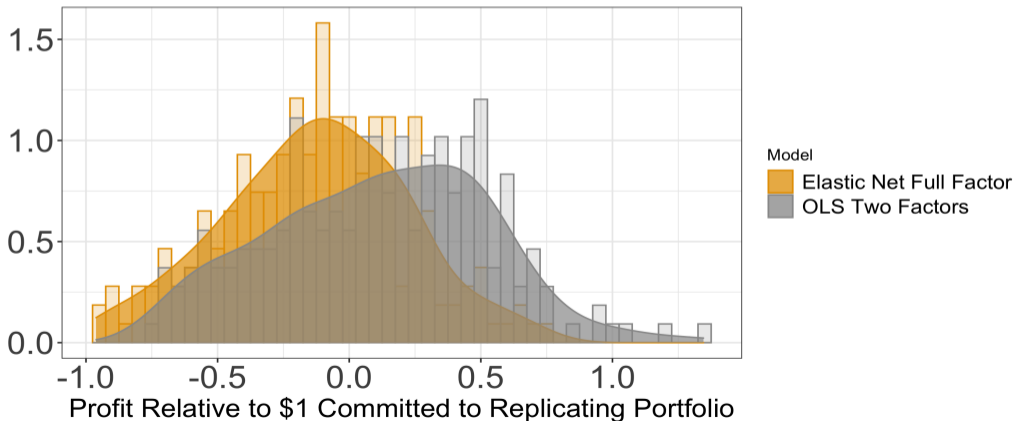


# PE Fund Risk-Adjusted Profits – Real Estate, Elastic Net

Back

## Histogram of Fund-Level Profit Relative to Replicating Portfolio

TVPI is:0.175 Risk-Adj Profit is:-0.154 Risk Adj Fraction above 10% is:0.259

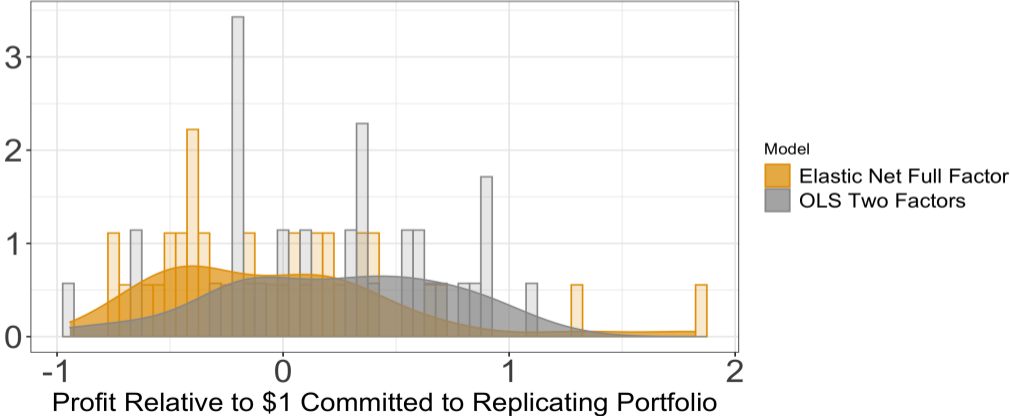


# PE Fund Risk-Adjusted Profits – Infrastructure, Elastic Net

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## Histogram of Fund-Level Profit Relative to Replicating Portfolio

TVPI is:0.17 Risk-Adj Profit is:-0.062 Risk Adj Fraction above 10% is:0.378

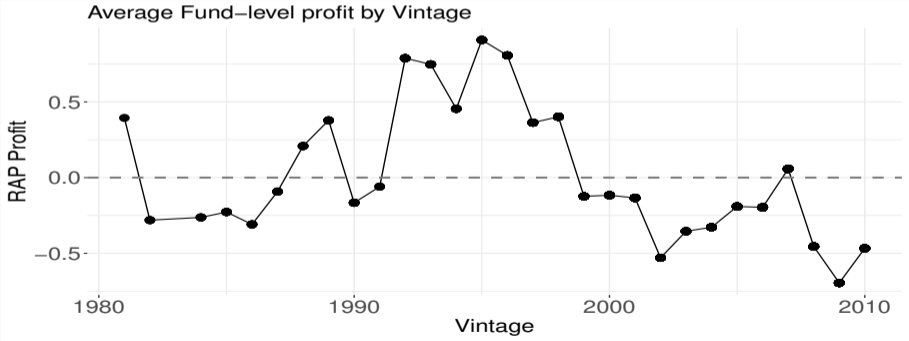


# PE Fund Risk-Adjusted Profits – VC, Elastic Net

Buyout    Venture Capital    Real Estate    Infrastructure

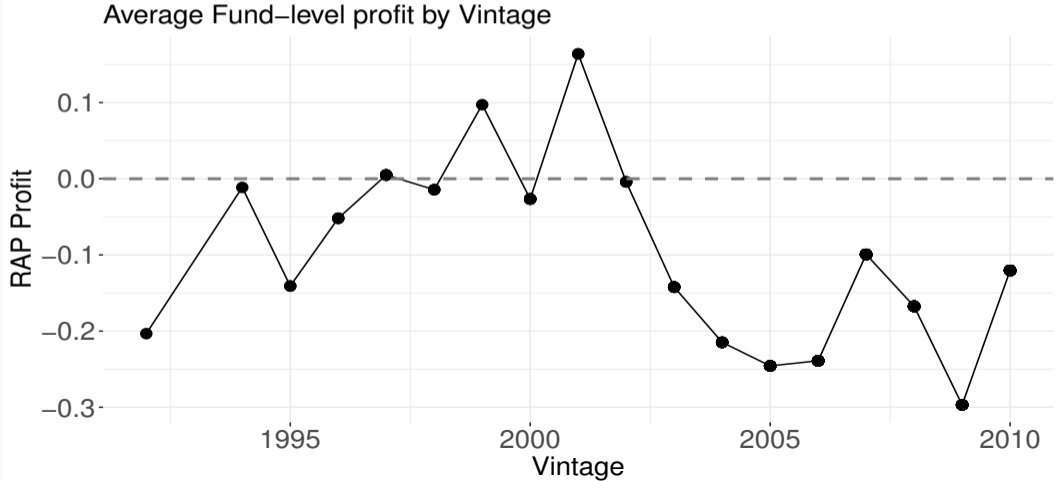
Buyout - Elastic Net    Venture Capital - Elastic Net    Real Estate - Elastic Net    Infrastructure - Elastic Net

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# PE Fund Risk-Adjusted Profits – Real Estate, Elastic Net

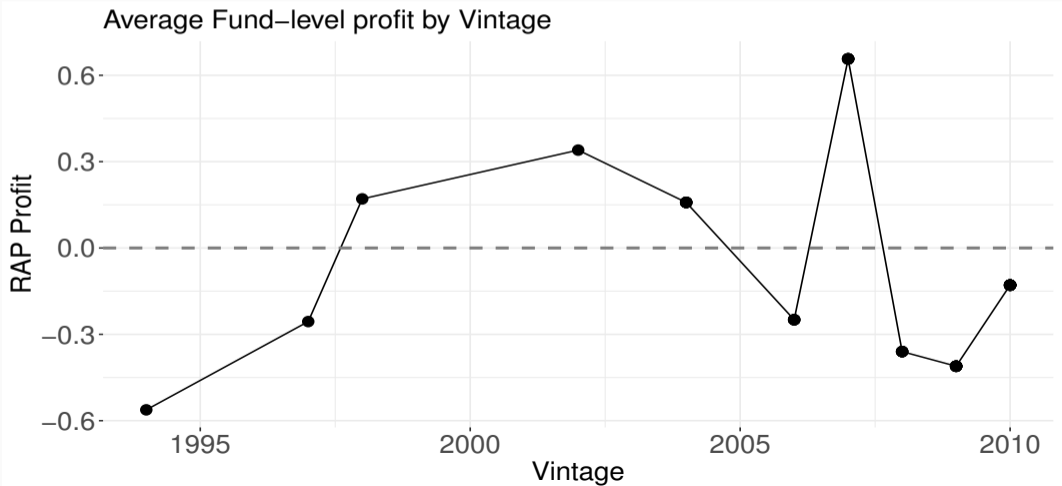
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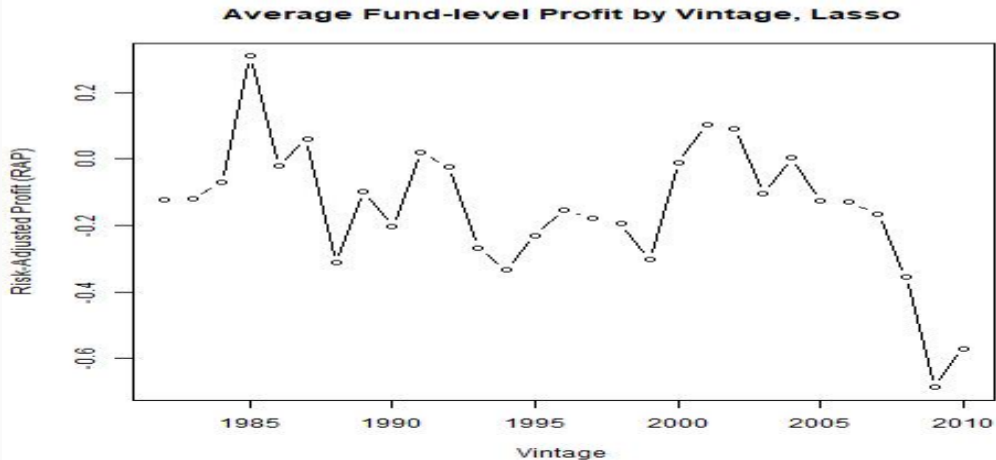
# PE Fund Risk-Adjusted Profits – Infrastructure, Elastic Net

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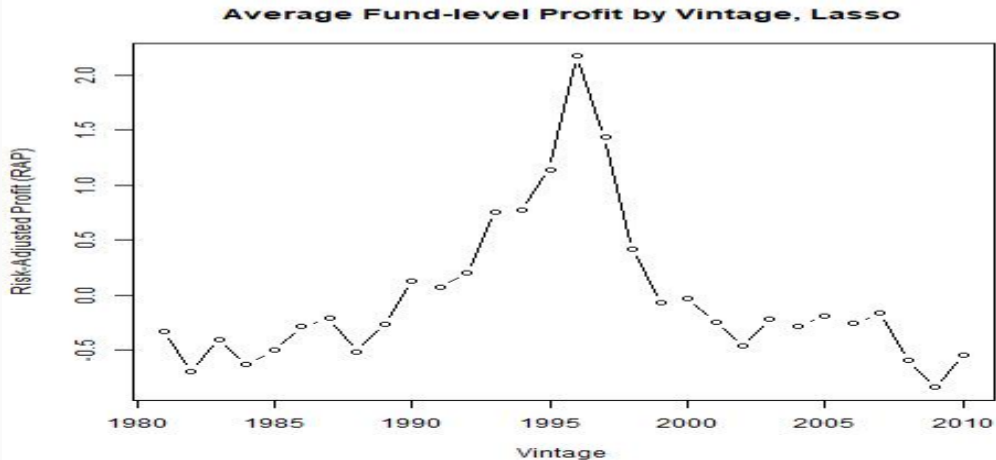
# PE Fund Risk-Adjusted Profits – Burgiss Buyout, Elastic Net

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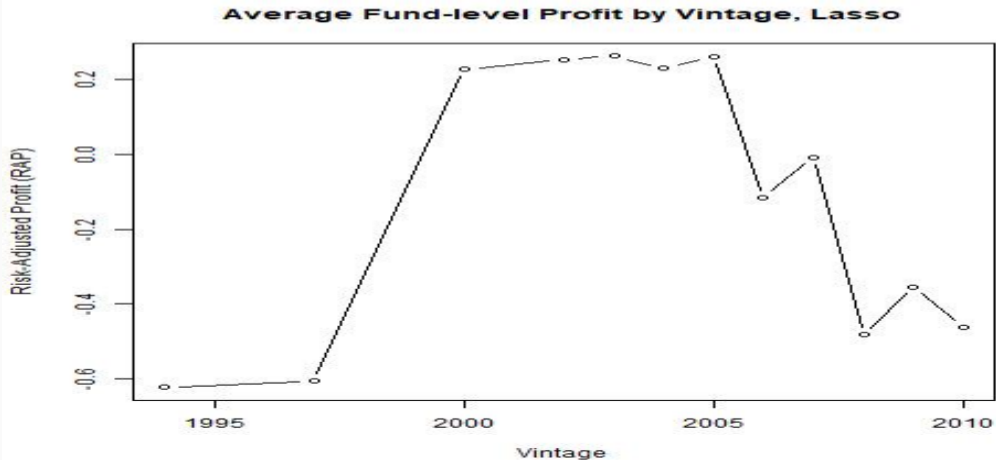
# PE Fund Risk-Adjusted Profits – Burgiss VC, Elastic Net

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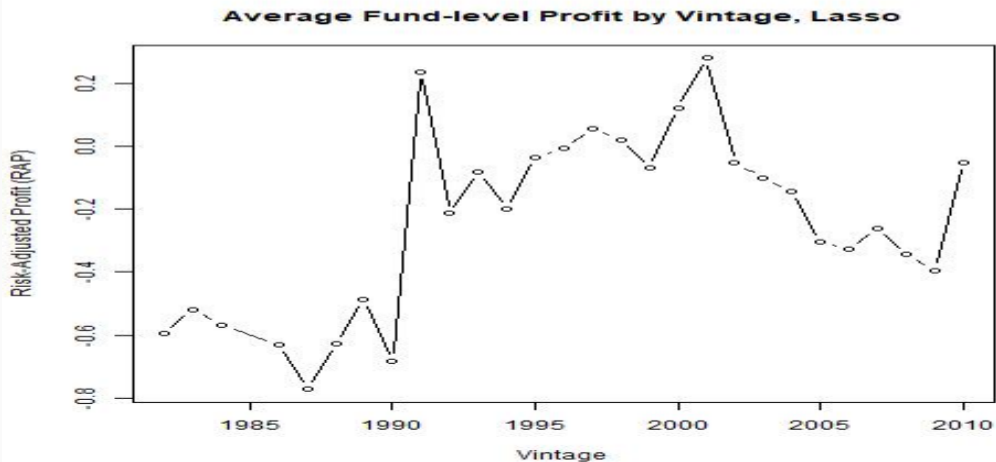
# PE Fund Risk-Adjusted Profits – Burgiss Real Estate, Elastic Net

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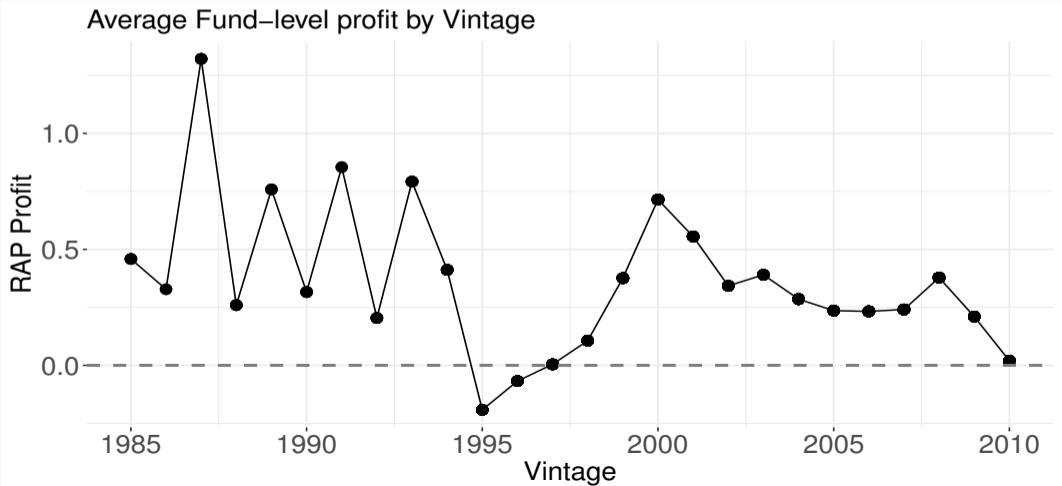
# PE Fund Risk-Adjusted Profits – Burgiss Infrastructure, Elastic Net

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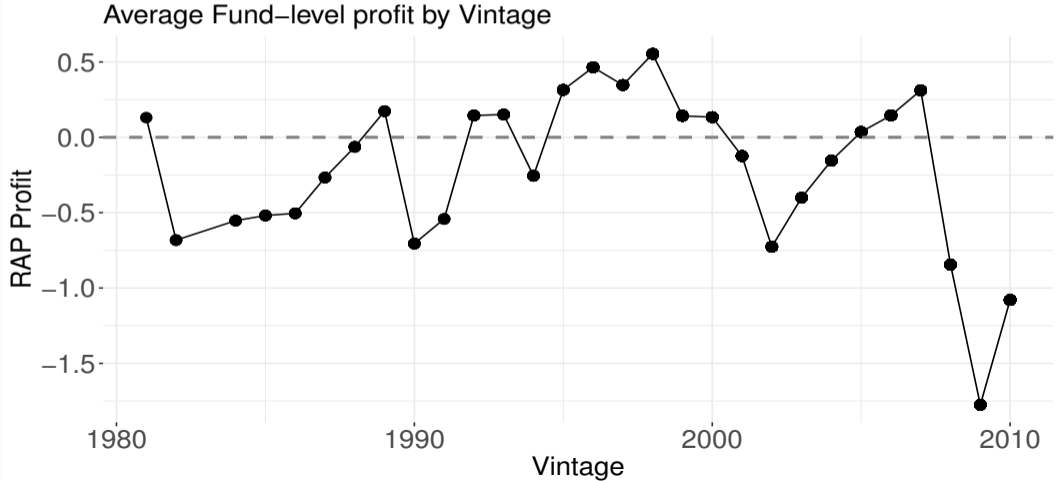
# PE Fund Risk-Adjusted Profits – Buyout, OLS

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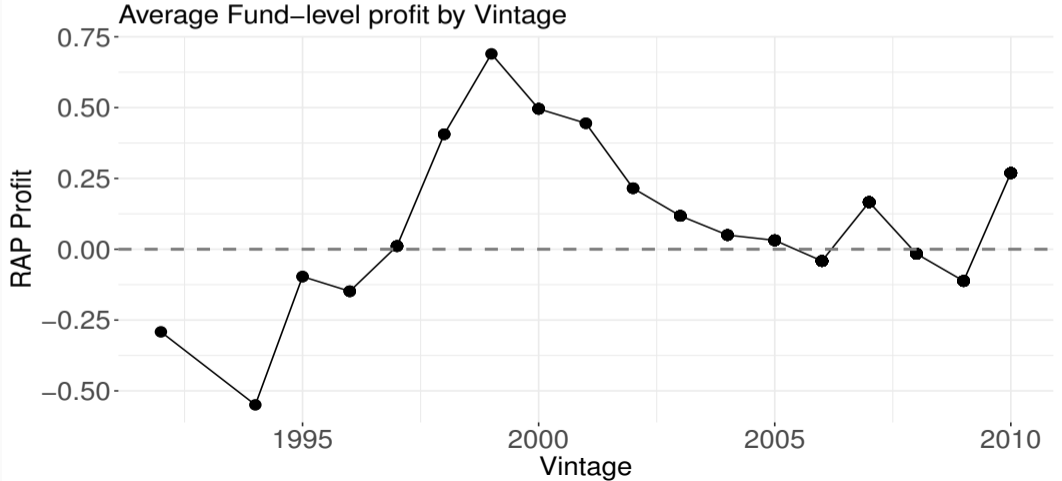
# PE Fund Risk-Adjusted Profits – VC, OLS

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# PE Fund Risk-Adjusted Profits – Real Estate, OLS

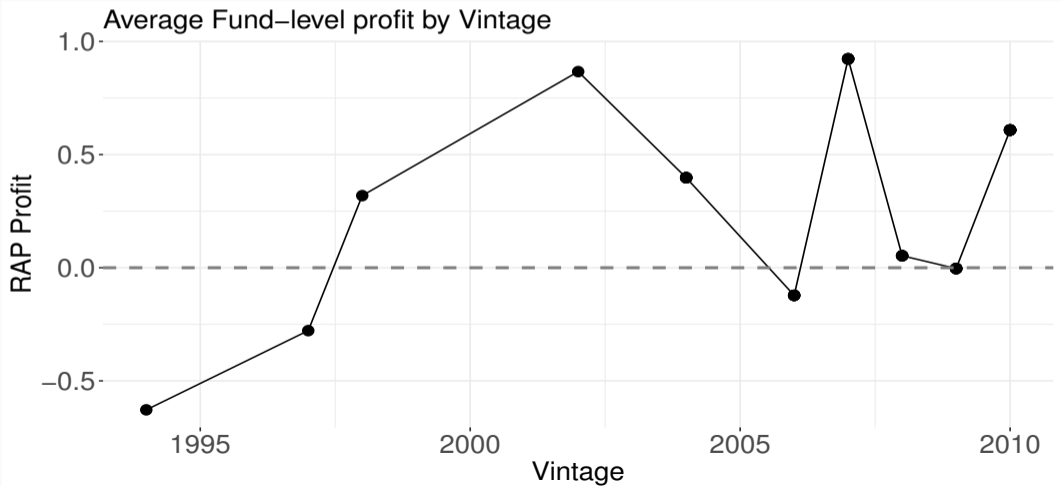
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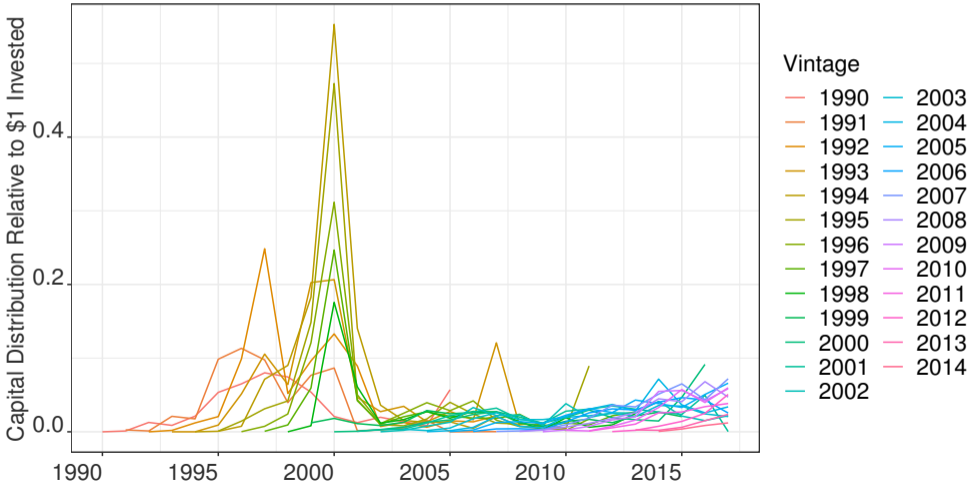
# PE Fund Risk-Adjusted Profits – Infrastructure, OLS

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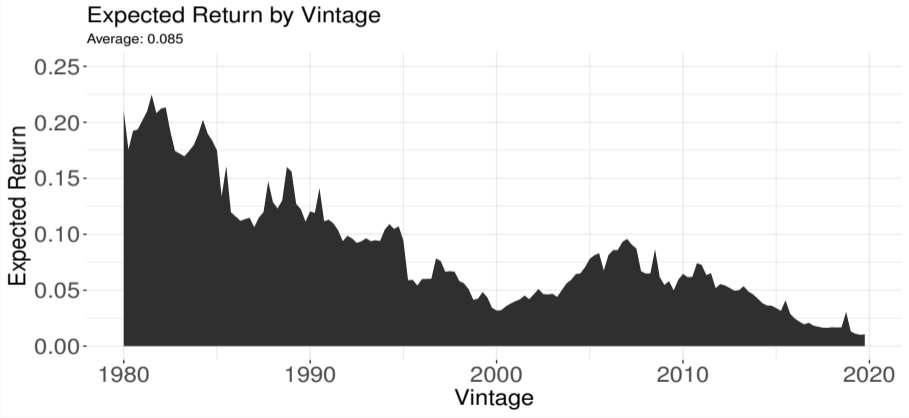
# Cash-Flow Variation Across Horizon and Vintage — Venture Capital

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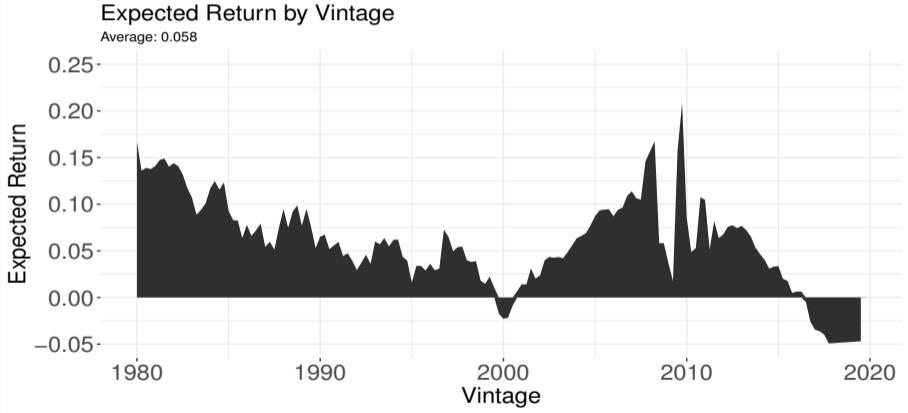
# PE Expected Return – Buyout, OLS

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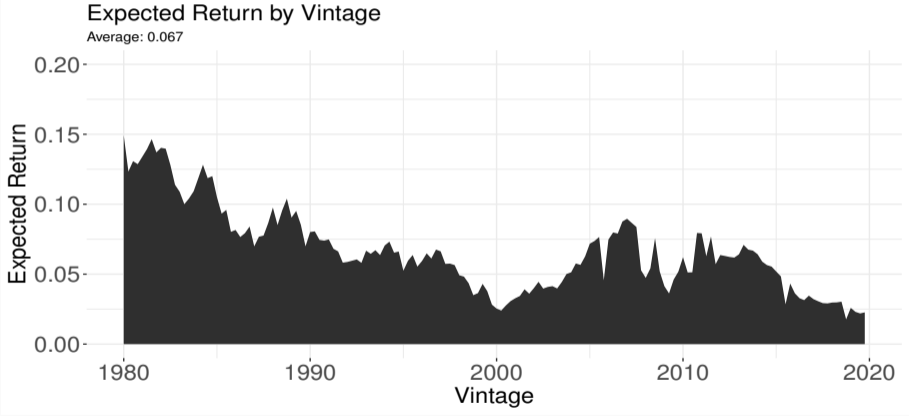
# PE Expected Return – VC, OLS

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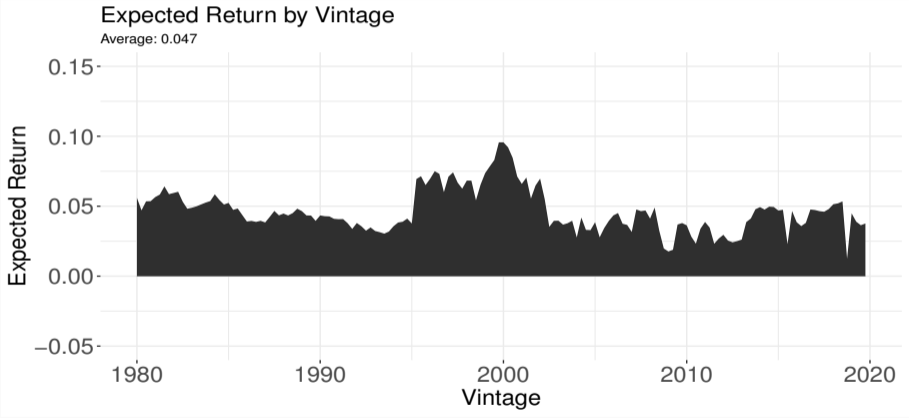
# PE Expected Return – Real Estate, OLS

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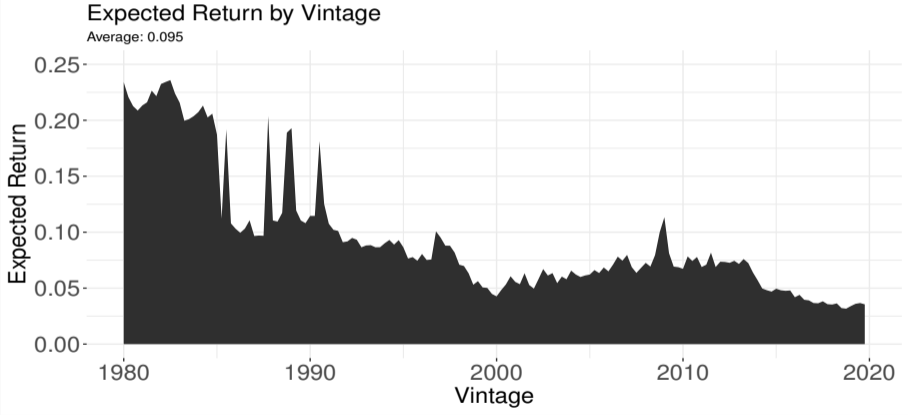
# PE Expected Return – Infrastructure, OLS

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# PE Expected Return – Buyout, Elastic Net

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# PE Expected Return – VC, Elastic Net

Buyout

Venture Capital

Real Estate

Infrastructure

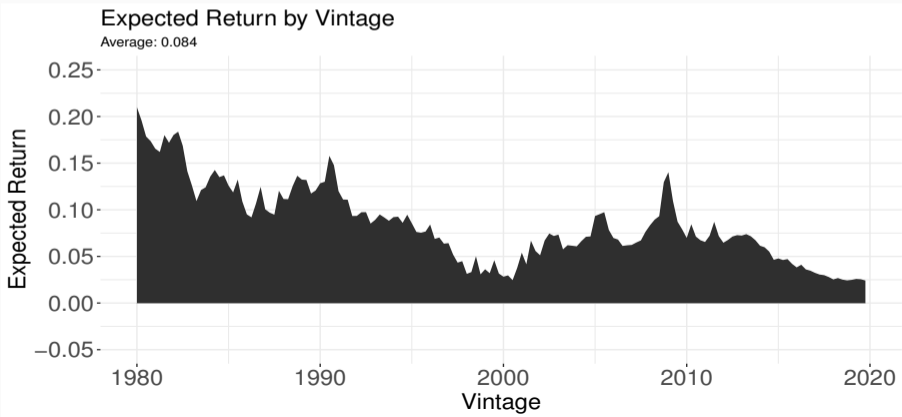
Buyout - Elastic Net

Venture Capital - Elastic Net

Real Estate - Elastic Net

Infrastructure - Elastic Net

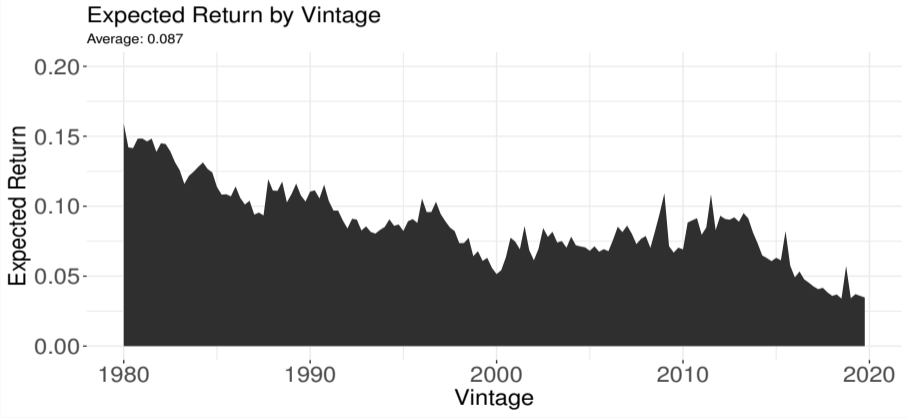
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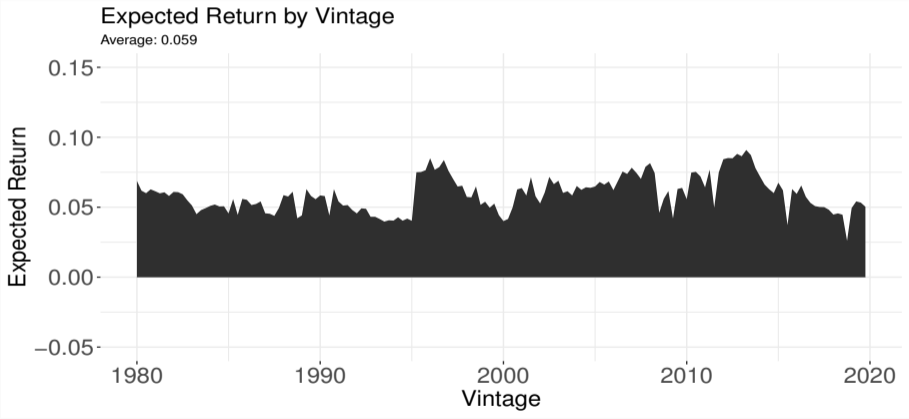
# PE Expected Return – Real Estate, Elastic Net

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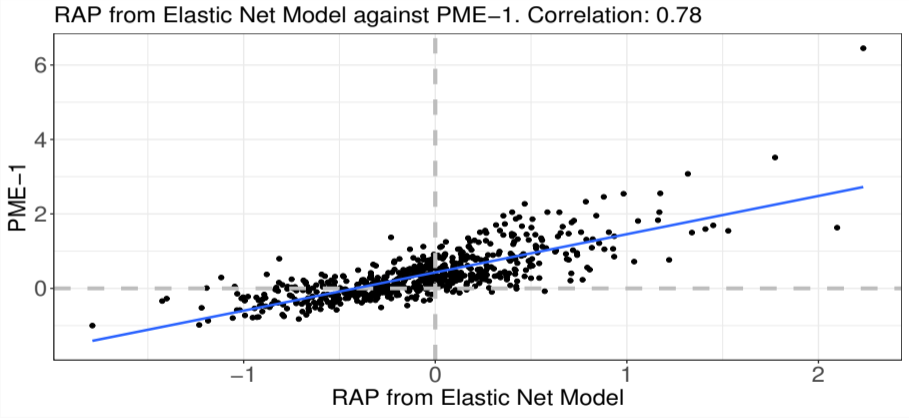
# PE Expected Return – Infrastructure, Elastic Net

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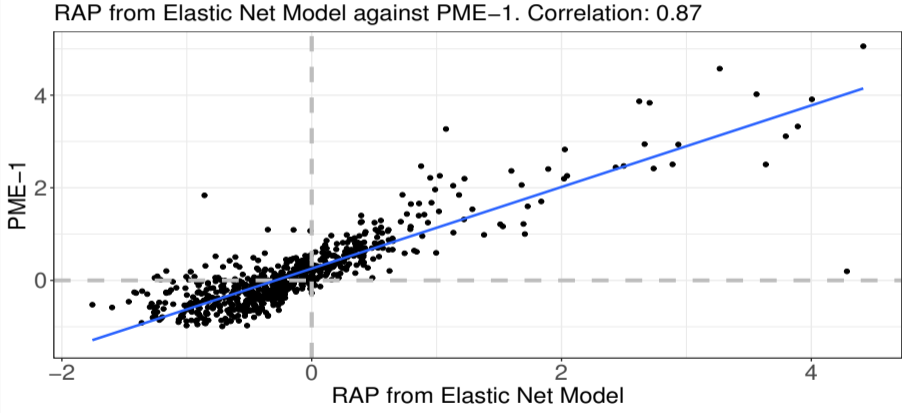
# PE Comparison with PME – Buyout

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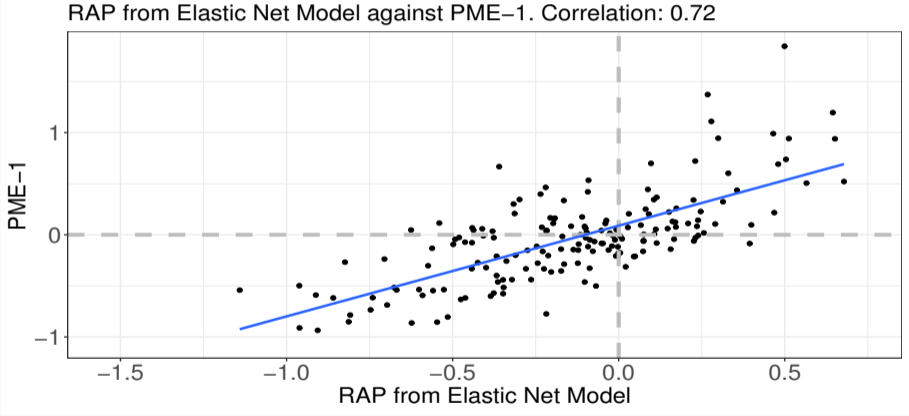
# PE Comparison with PME – VC

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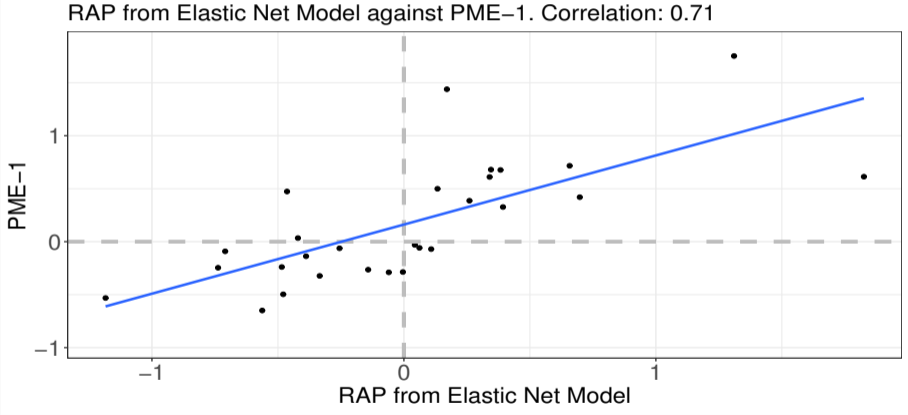
# PE Comparison with PME – Real Estate

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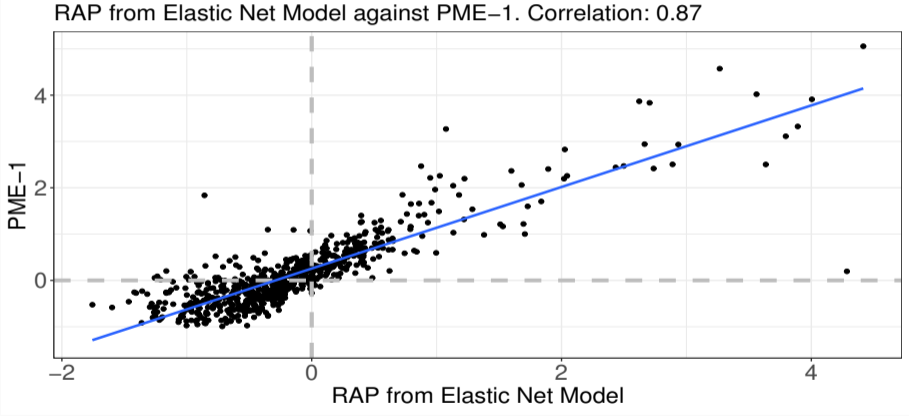
# PE Comparison with PME – Infrastructure

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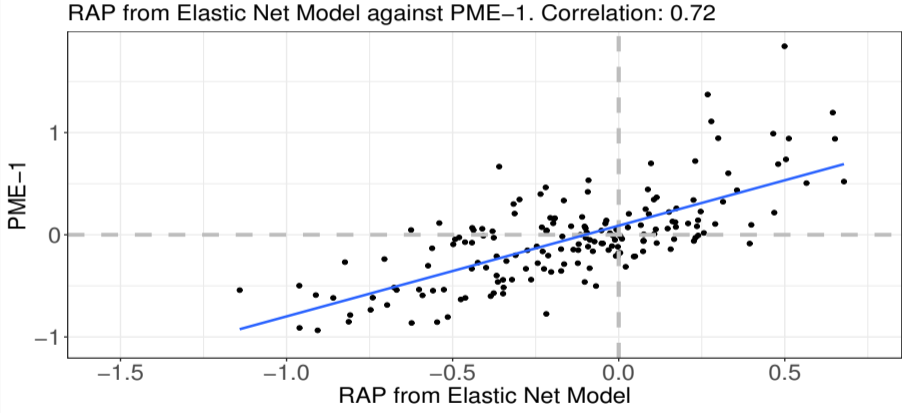
# PE Comparison with PME – VC, Elastic Net

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# PE Comparison with PME – Real, Elastic Net Estate

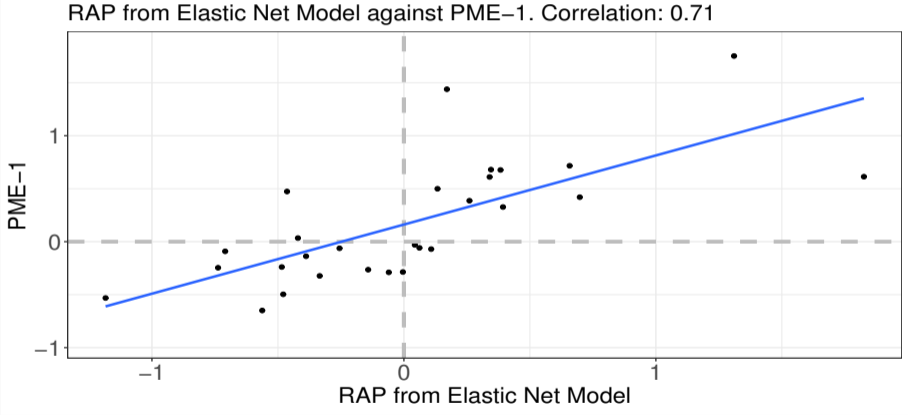
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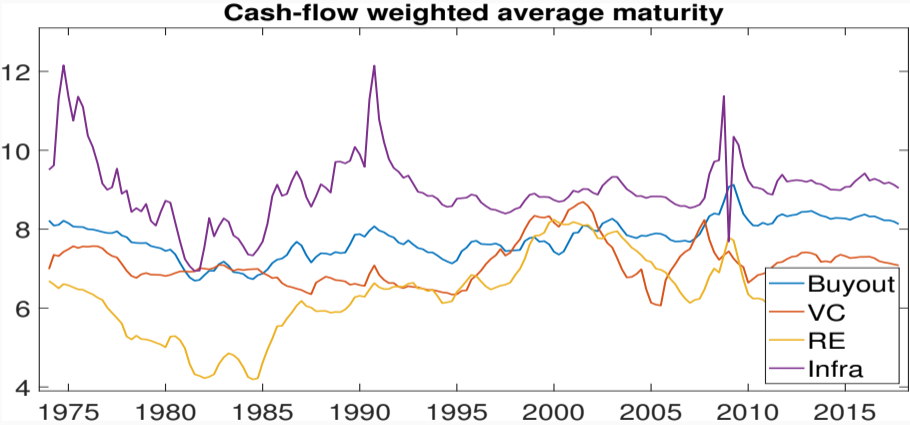


# PE Comparison with PME – Infrastructure, Elastic Net

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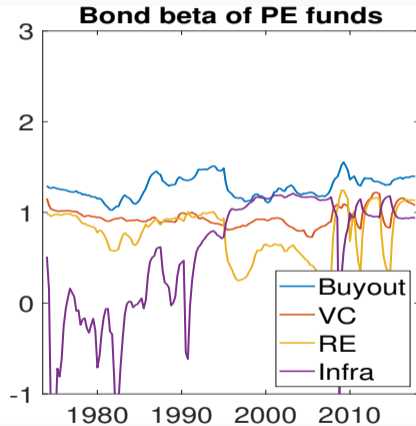
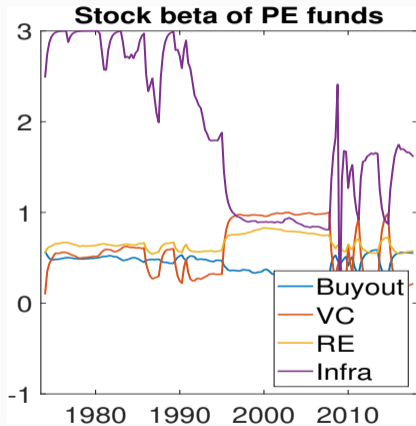


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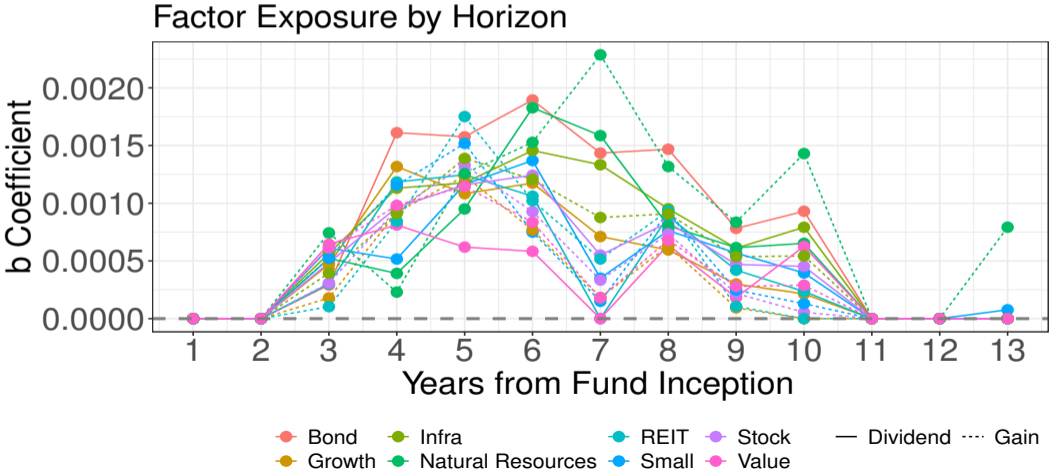
# Implied Stock and Bond Betas

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# Factor Exposure in PE Funds by Horizon – Infrastructure, Elastic Net

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# PE Comparison with IRR – Buyout

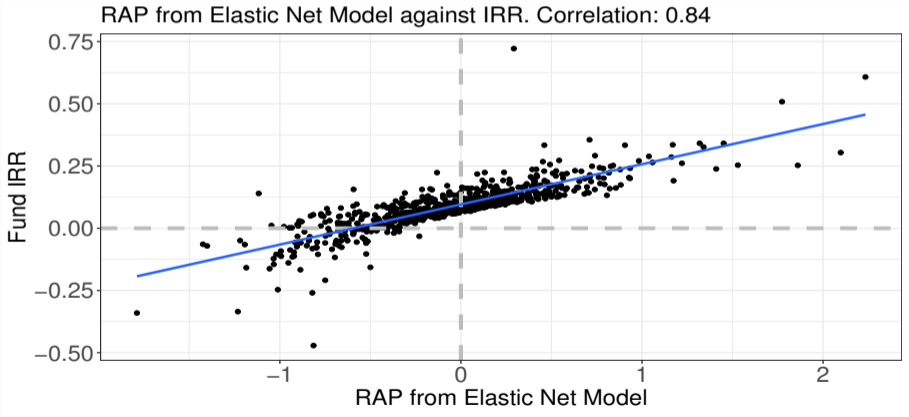
Buyout - Elastic Net

Venture Capital - Elastic Net

Real Estate - Elastic Net

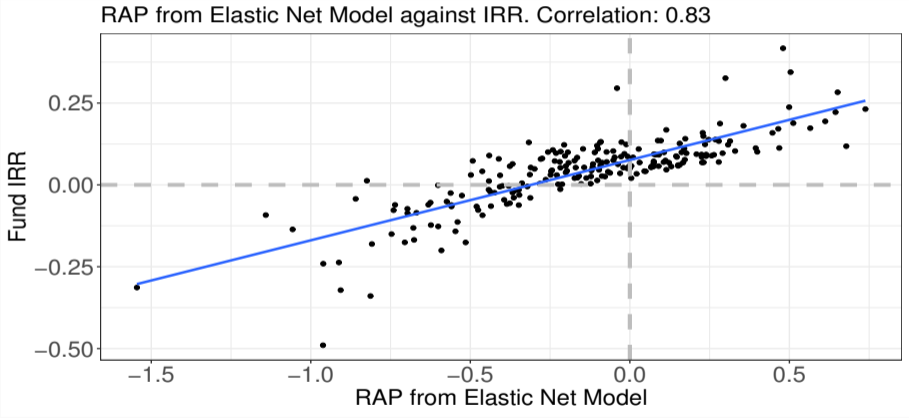
Infrastructure - Elastic Net

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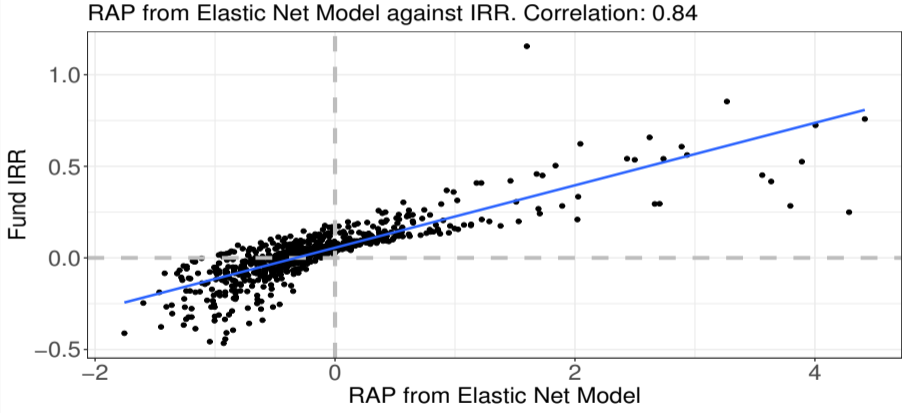
# PE Comparison with IRR – Real Estate

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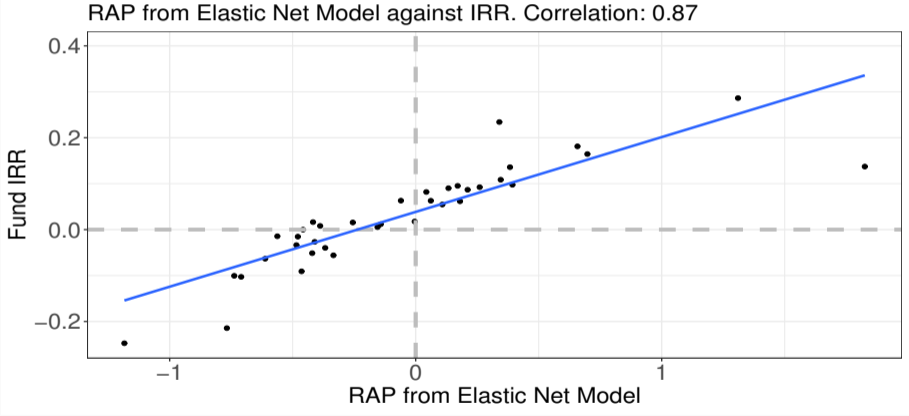
# PE Comparison with IRR – VC

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# PE Comparison with IRR – Infrastructure

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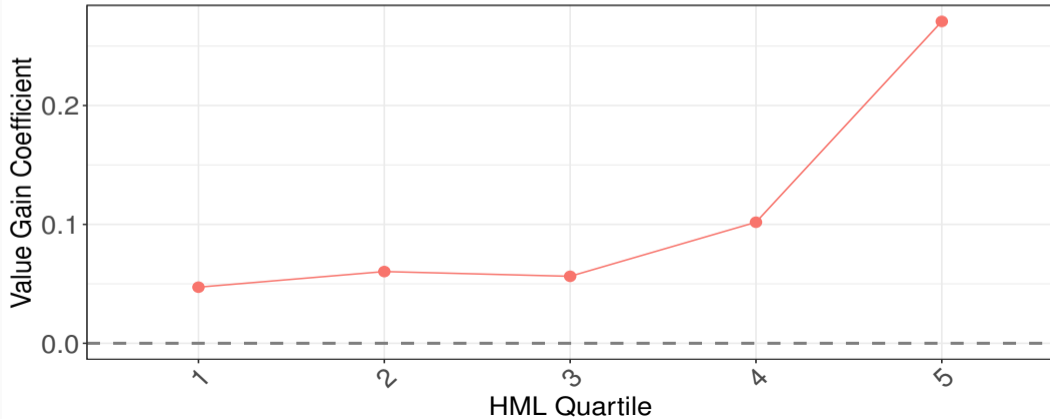


# HML Public Equity

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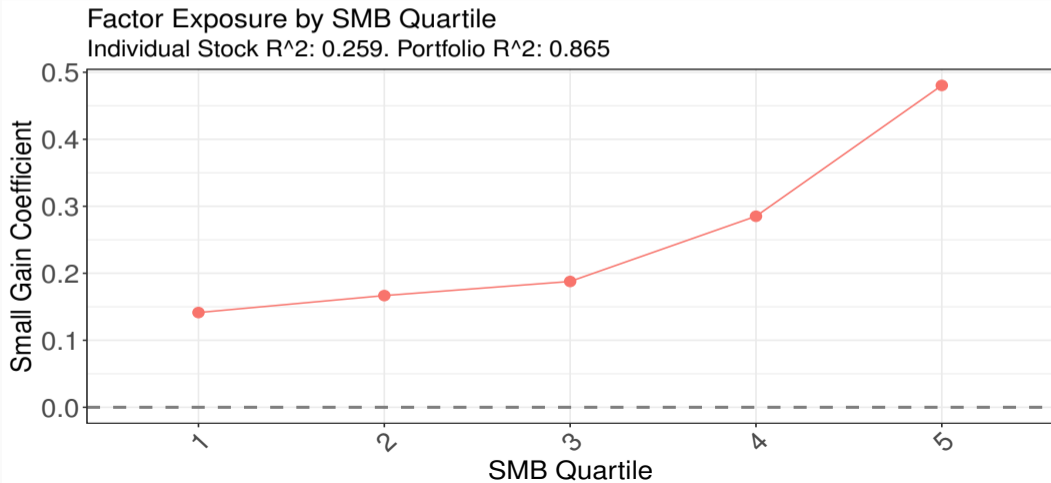
## Factor Exposure by HML Quartile

Individual Stock  $R^2$ : 0.258. Portfolio  $R^2$ : 0.88



# SMB Public Equity

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# MKT Public Equity

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